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Rational Homotopy Theory (Master)
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Sheet 5

Question 5.1

In Example 3.6 and Remark 3.7 we have defined two model structures on dgMod_R, the injective and the projective one. Show that the identity functor defines a Quillen equivalence beween the two model structures.

Question 5.2

Determine the following homotopy cofibers in the model category Top:

1. $S^2 \to *$,
2. $S^1 \to S^2$ by inclusion of the equator,
3. $S^1 \times S^1 \to S^1$ by projection to the first factor.

Question 5.3

Determine the homotopy colimit of the diagram $* \leftarrow * \amalg * \to *$ in Top.

Question 5.4

We work in the model category dgMod_R (feel free to use $\mathsf{dgMod}_{\mathbb{Q}}$ if you prefer).

Let M be a vector space and consider the cochain complex $M[i]$ which is M in degree $-i$ and 0 elsewhere.

What is the homotopy cofiber of $M[i] \to 0$?

What is the homotopy fiber of $0 \to M[i]$?

Hint: The model category of cochain complexes is *proper*, so it suffices to replace one map in the pushout (resp. pullback) diagram by a cofibraiton (resp. fibration).