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Rational Homotopy Theory (Master)
Winter Semester 2020/21

Sheet 3

Question 3.1

Find all generators up to degree 4 of the minimal model for $(S^2 \times S^3)\#(S^3 \times S^2)$.

Hint: The de Rham algebra contains forms x_2, y_2, x_3, y_3 in degree 2 and 3 which satisfy $x_2.x_3 = -y_2.y_3$ and span the de Rham cohomology.

Question 3.2

We defined a path object $\Omega(1)\tilde{\otimes}B \to B$ for augmented cdga's in lectures. Define the two maps $\Omega(1)\tilde{\otimes}B \to B$ corresponding to $\{0\} \to I$ and $\{1\} \to I$.

Question 3.3

We have seen in exmples that there is a model structure on chain complexes over $\mathbb{Q}$. Given two chain complexes A and B, can you give a criterion when there is a quasi-isomorphism $A \to B$?

Question 3.4

Show an example of two objects in $\mathsf{dgMod}_{\mathbb{Z}}^{\leq 0}$ such that there is a quasi-isomorphism $A \to B$ but no quasi-isomorphism $B \to A$.

Question 3.5

Show that in any model category cofibrations are stable under pushout, coproducts and transfinite composition.

These questions will be discussed in the exercise class on 23.11.20.

Questions with an asterisk are more challenging.