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Rational Homotopy Theory (Master)
Winter Semester 2020/21

# Sheet 12

## Question 12.1

Let $A$ be a $k$-algebar and $N$ a left $A$-module. Show that $B(A, A, N)$ is a free resolution of $N$.

*Hint:* To show this is a resolution define maps $\epsilon : B(A, A, N) \to N$ given by the action of $A$ on $N$, and $\iota : N \to B(A, A, N)$ given by $n \mapsto 1 \otimes n$. Now find a homotopy $s$ from $\iota \circ \epsilon$ to $\mathbf{1}_{B(A,A,N)}$.

## Question 12.2

Write down the Eilenberg-Moore spectral sequence for the Hopf fibrations. (You may use your knowledge of all the cohomology groups involved!)

Which differentials are nonzero?

## Question 12.3

Write down the long exact sequence of homotopy groups associated to the Hopf fibration.

## Question 12.4

Show that all homotopy groups of $S^\infty = \operatorname{colim}_n S^n$ are trivial.

## Question 12.5

Construct a $K(\mathbb{Z}/7, 1)$.

## Question 12.6

Is $S^1 \vee S^2$ nilpotent?

**These questions will be discussed in the exercise class on 8.2.20.**

Questions with an asterisk are more challenging.