Julian Holstein
Rational Homotopy Theory (Master)
Winter Semester 2020/21

Sheet 10

Question 10.1

Show that any map f in $\mathsf{dgMod}_{\mathbb{Q}}^{\geq 0}$ that induces 0 on cohomology is in fact homotopic to 0.

Question 10.2

Show that $\mathbb{Q} \oplus \mathbb{Q}.x$ with the comultiplication $1 \mapsto 1 \otimes 1$ and $x \mapsto x \otimes x$ is a coalgebra, satisfying axioms dual to those for an algebra.

Show that $\mathbb{Q}[x]$, considered as a vector space with basis $\{1, x, x^2, x^3, \dots\}$, has a coalgebra strucure whose comultiplication sends x to $x \otimes 1 + 1 \otimes x$.

The coalgebra structures on $C = \mathbb{Q} \oplus \mathbb{Q}.x$ and $C = \mathbb{Q}[x]$ induces natural algebra structures on $\mathrm{Hom}_{\mathbb{Q}}(C, \mathbb{Q})$. What are these algebras?

Question 10.3

Prove the 5-Lemma with a spectral sequence argument similar to what we did for the snake lemma in lectures.

Question 10.4 (*)

Let A, B be abelian groups. To compute $\mathrm{Ext}^1_{\mathbb{Z}}(A, B)$ you can resolve A by free abelian groups or B by injective abelian groups.

Why do the two methods give the same result?

Replace $\mathbb{Z}$ by a commutative ring R, A and B by R-modules and 1 by i for a more general result.

These questions will be discussed in the exercise class on 25.1.20.

Questions with an asterisk are more challenging.