Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Julian Holstein
Algebraic Topology (Master)
Summer Semester 2023

Exercise sheet 8

Question 8.1

Let A and B be finitely generated abelian groups. Determine the homology groups of the product of the Moore spaces $M(A,n)$ and $M(B,m)$, $H_*(M(A,n) \times M(B,m))$, for arbitrary natural numbers $n, m \geq 1$.

Question 8.2

Compute the homology of $\mathbb{R}P^2 \times \mathbb{R}P^3$ with coefficients in $\mathbb{Z}$ and with coefficients in $\mathbb{Z}/2$.

Question 8.3

Use a suitable simplicial complex to compute the homology of S^2.

Question 8.4

A Δ-complex is a generalization of a simplicial complex. A Δ-complex consists of collections K_n of n-simplices for all $n \geq 0$ together with face maps $d_i : K_n \to K_{n-1}$ for $i = 0, \ldots, n$, which satisfy $d_i \circ d_j = d_{j-1} \circ d_i$ whenever $i < j$.

The main difference to a simplicial complex is that a simplex is no longer determined by its vertices.

For a Δ-complex K we may define $|K|$ as the quotient of $\cup_n K_n \times \Delta^n$ where we identify $(d_i\sigma, t) \simeq (\sigma, d^i t)$ where $d^i : \Delta^n \to \Delta^{n+1}$ is the inclusion of the i-th face.

Simplicial homology for a Δ-complex is defined in the same way as for a simplicial complex, with $C_i = \oplus_{K_i}\mathbb{Z}$ and $d = \sum(-1)^i d_i$.

a) Represent the circle as a Δ-complex K with two 1-simplices and two 0-simplices. Verify that $|K|$ constructed as above is S^1. Compute the simplicial homology of K.

b) Do the same for the sphere S^2.

c) Represent the torus and the Klein bottle as Δ-complexes with two 2-simplices and compute their simplicial homology.

These questions will be discussed in the class on 7/6/2023. You may hand in your solutions the day before.

Questions with an asterisk are more challenging.