A general shape optimization problem:

$$\begin{cases} \min\ J(u(\Omega),\Omega) \\ \text{s.t.}\ \ \Omega \in \mathcal{O} \\ \mathcal{F}(u(\Omega)) = 0 \quad \text{in } \Omega \end{cases}$$

$u(\Omega), \Omega$: state and control variable
J: cost functional
$\mathcal{O}$: family of admissible domains
$\mathcal{F}$: formal description of the state equation

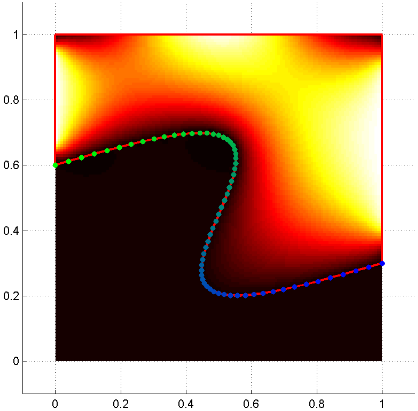
Preconditions: In this research project the family $\mathcal{O}$ is described by a set of admissible parameters $\mathcal{S}$, i.e. $\mathcal{O} := \{\Omega_\gamma \subset \mathbb{R}^2 : \gamma \in \mathcal{S}\}$. The boundary of an admissible domain Ω_γ is given by $\partial\Omega_\gamma = \overline{\Gamma} \cup \overline{\Gamma}_\gamma$, where Γ is a fixed and Γ_γ is a variable boundary part. More precisely Γ_γ is the image of an appropriate curve $\gamma : (0,1) \to \mathbb{R}^2$, $\gamma \in \mathcal{S}$.
Ansatz: All admissible domains Ω_γ are embedded in a fixed domain $\hat{\Omega}$, i.e. $\hat{\Omega} \supset \Omega_\gamma$ for all $\gamma \in \mathcal{S}$. By means of a Lagrange multiplier g_γ this Ansatz leads to an equivalent problem formulation, where the state eqaution is given on the fixed domain $\hat{\Omega}$.

$$\begin{cases} \min\ J(u_\gamma,\gamma) \\ \text{s.t.}\ \ \gamma \in \mathcal{S} \\ \mathcal{F}(u_\gamma) = 0 \quad \text{in } \Omega_\gamma \end{cases} \iff \begin{cases} \min\ J(\hat{u}_\gamma|_{\Omega_\gamma},\gamma) \\ \text{s.t.}\ \ \gamma \in \mathcal{S} \\ \hat{\mathcal{F}}(\hat{u}_\gamma, g_\gamma) = 0 \quad \text{in } \hat{\Omega} \end{cases}$$

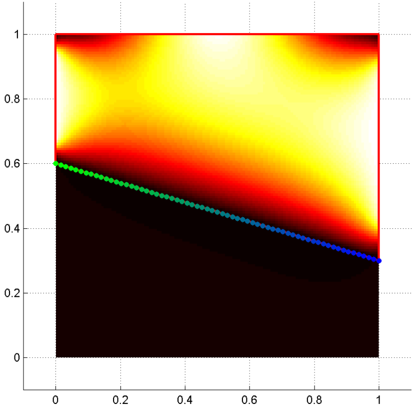
For a numerical example we use:

- a cost functional of tracking type
- the Poisson equation with Dirichlet boundary conditions as the state equation.
- a Gradient descent method with Armijo rule.

Optimization process: We try to track the desire control and state you see on the left picture. On the right side you see the initial control and state. Click on the right image to start an animation.



Desired state



Iteration: 0