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Attention of students during mathematics lectures

Introduction

In teaching mathematics in a classroom, some of the most basic (and most important) questions are:

1. To what extent do students pay attention?
2. To what extent do they understand the content?
3. What can be done to improve their understanding?

Here we focus on the first question (whether students pay attention), since addressing the other questions requires to first address the first question.

Despite the fundamental importance of these questions, the literature contains too few answers. The topic of attention during lectures has been studied for decades (and recent debates in the literature are still controversial). The topic of attention specifically during *mathematics* lectures, on the other hand, has not received much attention. This article discusses previous approaches and presents a new approach in an attempt to gain new insight.

The "classical" literature on student attention

One of the most cited results concerning attention of students is the work of Johnstone and Percival (1976), who studied attention of students during lectures in large classrooms (300 or more students). The lectures were on chemistry. (This raises the question of the extent to which their results apply to mathematics lectures; no similar study in mathematics lectures is known to the author.) In their study, observers placed in the classroom recorded the times during the lecture when the majority of the audience showed "general lack of concentration", measured by background noise and "doodling, looking idly around, yawning, chatting, etc." The observers also recorded which topic was covered at that time. Johnstone and Percival show that a group of students displaying such "lack of concentration" perform worse at exam questions dealing with the topic covered than a control group of students who seemed more attentive while being taught the same topic. This shows (so the authors thought) that attention (as a relevant factor to understanding) of students can be easily observed by outsiders present during the lecture. Johnstone and Percival also investigated at what times during a lecture these periods of inattention occured. (Their findings are: briefly at the beginning, then after 10 to 18 minutes, then increasingly often, and approximately every 3 minutes near the end of a 50-minute lecture.) They mention that this depends on the lecturer. They moreover as-

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sume that it depends on difficulty, speed, legibility of writing, and the personality of the lecturer, but do not verify these assumptions. They give some suggestions for avoiding inattention (take breaks; change between theory and experiment; show models; solve problems). They also noted that students who were forced to view a video transmission of the lecture instead of being allowed to enter the (overflowing) lecture hall showed decreased attention. (Whether or not the latter still applies with today's transmission technology might be an interesting question to study.)

Many other articles followed Johnstone and Percival's assumption that the attention decrease can be easily and accurately measured. Based on this assumption, they tried to further analyze the decrease. See e.g. Hartley and Cameron (1967), Maddox and Hoole (1975).

Modern literature on student attention

The modern literature is critical of several of the aforementioned results and conclusions. Szpunar, Moulton and Schacter (2013) argue that visible signs of attention (or inattention) are not a good indicator of actual attention (citing Wilson & Korn, 2007), and note-taking may not be suitable either to assess attention (McClendon, 1958; Maddox & Hoole, 1975). Szpunar et al. state that psychologists understand attention well (from a cognitive and neural point of view), and that educators would greatly benefit from understanding this, but that no one has ever brought these two areas of knowledge together. They moreover indicate that there are only very few actually valid studies of attention in lectures; in particular, they are critical of Johnstone and Percival's results. But it remains unclear how attention is supposed to be measured correctly.

Mathematics-specific study of attention

The author of this article suggests to specifically study attention in *mathematics* courses. It seems plausible that patterns of attention which are optimal for understanding mathematics courses are different from patterns of attention optimal for courses in other fields (and that the latter patterns differ also). For example, in mathematics, sequential dependence is extremely high; in other words, students who fail to understand one concept (definition, equation, or theorem) will have great difficulties whenever this concept is used again; and although the same applies to some degree to any field of study, the necessity to have completely understood preceding material is especially strong in mathematics. Hence the effect of attention during lectures on student understanding should be especially strong when those lectures are *mathematics* lectures.

An attempt to measure attention in mathematics courses

The author of this article text is attempting to measure attention in mathematics courses, or more precisely a concept which might be called *attention plus retention* (explained in the following), which is closer to understanding than mere *attention* is. The class chosen for this study was a course in undergraduate Analysis at PH Vorarlberg, Austria, for students about to become secondary school teachers. The class consisted of a weekly lecture, followed by a weekly recitation/exercise session one day later.

An approach similar to Johnstone and Percival's was out of the question, primarily due to the aforementioned fundamental problems of the approach, and also because their method (measuring substantial lack of concentration in the classroom) probably only works with large classes (300 students in their case). The classes studied here are much smaller (30 students or less), tend not to produce much measureable background noise, and tend to be well-behaved, which makes it difficult to determine whether or not students are paying attention.

A method is needed which is easy to use, does not use much of the students' time (unless time used thusly is somehow beneficial in other ways), requires little or no personnel other than the lecturer, and requires no psychological self-assessment from the students (such assessments in mathematics courses are highly unreliable). The new method presented here (still work in progress, suggestions for improvement are welcome) is: During the first minutes of the recitation/exercise session, an anonymous questionnaire is handed out containing a list of approximately 20 items (keywords, statements, and formulas) from the preceding lecture. Students are asked to indicate next to each of these items whether they do or do not remember the item from the lecture. (There is also the option "I am not sure" to avoid having to extrapolate from the number of wrong answers.) In order to prevent students from ticking the "yes" column blindly, the questionnaires also contain some items (mixed in with the others) which are actually not part of the preceding lecture. There is no motivation for students to intentionally give incorrect answers; it is pointed out to them that their answers on these (anonymous) questionnaires are for research purposes only and will not affect their course grades. Students are instructed to complete the questionnaires without much pondering in about 2 minutes, thus answering each of the approximately 20 questions in just a few seconds. (No time limit is actually enforced.) The students did not seem to mind filling out a questionnaire during each exercise class, presumably because it is quick. The keywords, statements and formulas are chosen to cover the entire time span of the preceding lecture and all of its mathematically relevant content.

One of the initial goals of this research was to detect some pattern of attention: Does attention drop after a certain number of minutes in class? How does attention depend on the topic? (How it depends on the lecturer and lecturing style is an interesting topic for future research.)

Preliminary results indicate (somewhat surprisingly) that the measured level of *attention plus retention* is very high. For almost all of the items almost all of the students could tell correctly whether they were covered in the previous class or not. The error rate is not much larger than what would be expected due to linguistic misunderstandings and ambiguities. E.g., the names *mean value theorem* and *intermediate value theorem* (German: *Mittelwertsatz, Zwischenwertsatz*) are probably too difficult to distinguish for students if the class has covered one of these theorems but not yet the other. Also, if a particular lecture covered the product rule for differentiation *(fg)' = f'g+fg'* and the explanation "in order to differentiate a product, it is incorrect to simply differentiate each factor", it is ambiguous whether the incorrect "equation" *(fg)' = f'g'* was "covered" in class.

Conclusions of this study so far seem to be: The main problems students may have understanding mathematics seem *not* to include basic attention or *attention plus retention*. *(*For the small class studied here; large classes, as the ones studied by Johnstone and Percival, *do* seem to have attention problems.) The focus of the questionnaires on mathematics-specific concepts (formulas, names of theorems) seems to be helpful. While the findings are too preliminary to make suggestion yet, class size and its effect on students' understanding of mathematics merits further research. Also, gathering more data that specifically covers *mathematics* lectures is needed.

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