# Analysis of High-Dimensional Signal Data by Manifold Learning and Convolution Transforms

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**Abstract**

Recent advances in nonlinear dimensionality reduction and manifold learning have provided novel methods in the analysis of high-dimensional signals. In this problem, a very large data set $U \subset \mathbb{R}^n$ of scattered points is given, where the data points are assumed to lie on a compact submanifold $\mathcal{M}$ of $\mathbb{R}^n$, i.e. $U \subset \mathcal{M} \subset \mathbb{R}^n$. Moreover, the dimension of $\mathcal{M}$ is assumed to be much smaller than the dimension of the ambient space $\mathbb{R}^n$, i.e. $\dim(\mathcal{M}) \ll n$. Now, the primary goal in the data analysis through dimensionality reduction is to construct a low-dimensional representation of $U$. The dimensional reduction map is required to preserve intrinsic geometrical and topological properties of the manifold $\mathcal{M}$ in order to obtain a sufficiently accurate (low-dimensional) approximation of $U$. In this project, we analyze the effects of combining convolutions filters (using in particular suitable wavelet transformations) with dimensionality reduction maps in order to improve the construction of low-dimensional representations. This task involves the understanding of the geometrical distortion caused by the convolution transform in the manifold $\mathcal{M}$. The properties of the resulting nonlinear dimensionality reduction method are illustrated by numerical examples concerning low-dimensional parametrization of scale modulated signals and solutions to the wave equation at varying initial conditions.

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