

Probabilistic Methods in Combinatorics

Exercise Sheet 1

Question 1. How many elementary events are there in $\mathcal{G}(4, 1/2)$? Calculate the following probabilities:

- $\mathbb{P}(G(4, 1/2) \text{ has 2 edges})$;
- $\mathbb{P}(G(4, 1/2) \text{ has 6 edges})$;
- $\mathbb{P}(G(4, 1/2) \text{ is connected})$.

Question 2. For which p is the probability measure on $G(n, p)$ the uniform distribution?

Question 3. Give an example of a set of events $\{A_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of a collection of random variables $\{X_i : i \in I\}$ which are pairwise independent, but not mutually independent.

Give an example of two random variables X and Y such that $\mathbb{E}(XY) \neq \mathbb{E}(X)\mathbb{E}(Y)$.

Question 4. Show that with high probability $\mathbb{P}(G(n, p) \text{ has diameter } \leq 2)$, for constant p .

Question 5. Let $T = (x_1, x_2, \dots, x_m)$ be a sequence of not necessarily distinct reals and let

$$T_b = \{(x_i, x_j) \mid |x_i - x_j| \leq b\}.$$

a) Show that $|T_2| < 3|T_1|$

b) Prove that for every two independent identically distributed real random variables X and Y ,

$$\mathbb{P}(|X - Y| \leq 2) \leq 3\mathbb{P}(|X - Y| \leq 1)$$