

Sightseeing im Hyperraum

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Hyperraum



JEDIPEDIA
Das Star Wars-Wiki

- ▶ “Außerhalb des Universums ist nichts. Dieses Nichts nennt man **Hyperraum.**”

Hyperraum



- ▶ “Der **Hyperraum** ist ein dem normalen Raum übergeordnetes Kontinuum, das überlichtschnelles Reisen ermöglicht. Bei der Passage präsentiert er sich dem Reisenden als stürmische und alptraumhaft rot glühende Umgebung, die keinerlei Referenzpunkte bietet.”

Hyperraum

Gerade \mathbb{R} 

Ebene \mathbb{R}^2 

Raum \mathbb{R}^3 

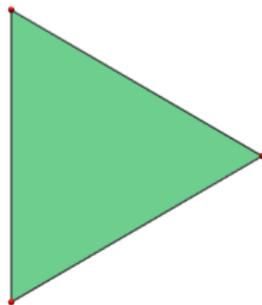
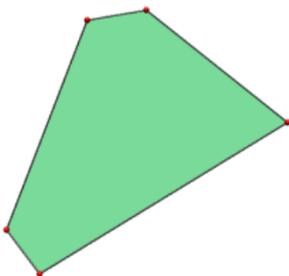
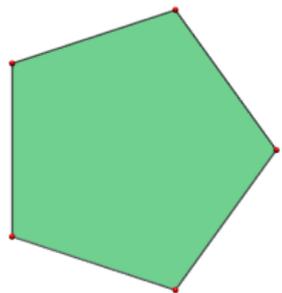
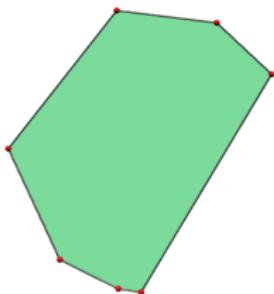
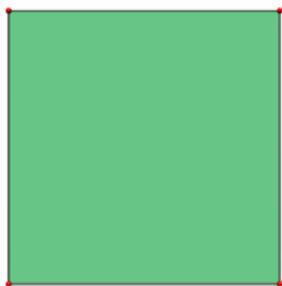
Hyperraum \mathbb{R}^4 

Polytope

- ▶ Wollen ein Gefühl für die verschiedenen Dimensionen entwickeln.
- ▶ Dazu untersuchen wir geometrische Figuren.
- ▶ Aber nur solche mit flachen Seiten, genannt **Polytope**...
- ▶ ...die mit jedem Paar von Punkten auch die Gerade dazwischen enthalten, genannt **konvexe Polytope**.

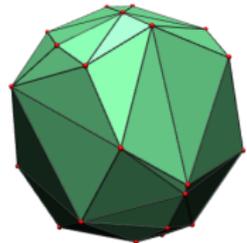
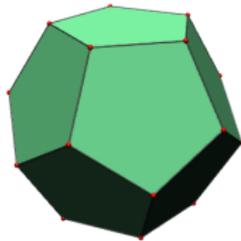
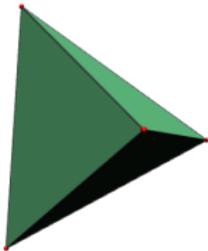
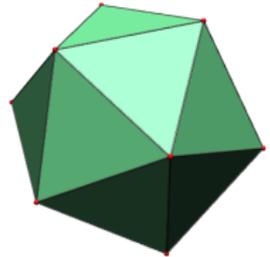
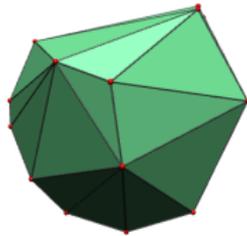
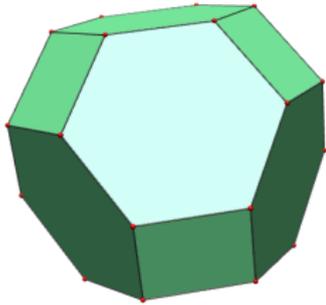
\mathbb{R}^2

► In der Ebene sind das konvexe **Polygone**:



\mathbb{R}^3

► Im Raum konvexe **Polyeder**:



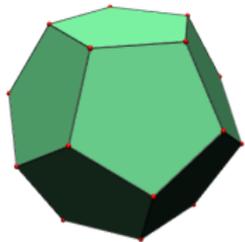
\mathbb{R}^4

- ▶ Wie soll man sich ein Polytop im Hyperraum vorstellen?

$$\mathbb{R}^3 \longleftrightarrow \mathbb{R}^2$$



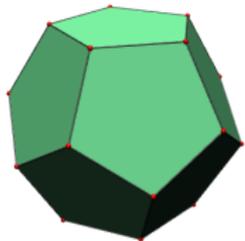
$$\mathbb{R}^3 \supset$$



$$\mathbb{R}^3 \longleftrightarrow \mathbb{R}^2$$



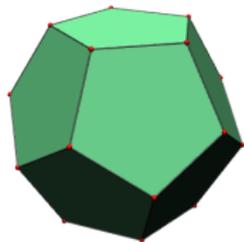
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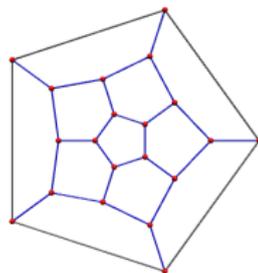
$$\mathbb{R}^3 \longleftrightarrow \mathbb{R}^2$$



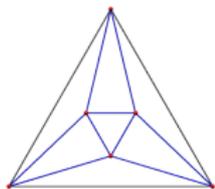
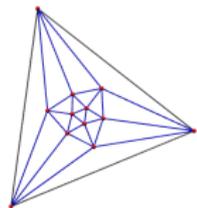
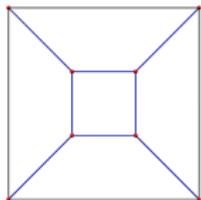
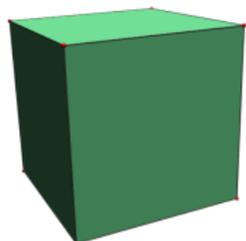
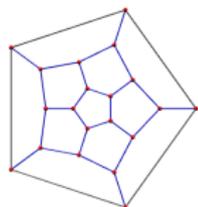
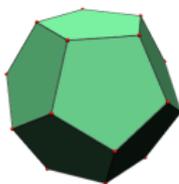
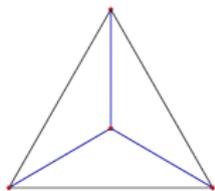
$$\mathbb{R}^3 \supset$$



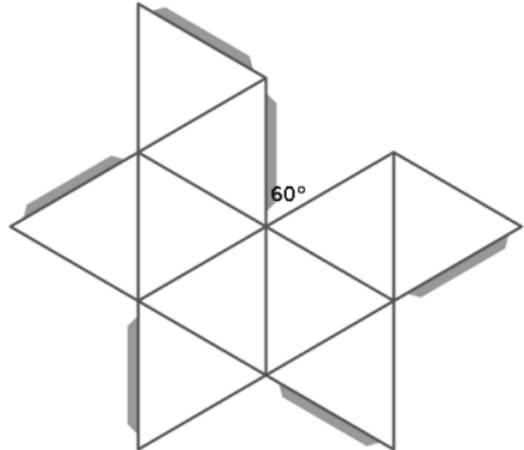
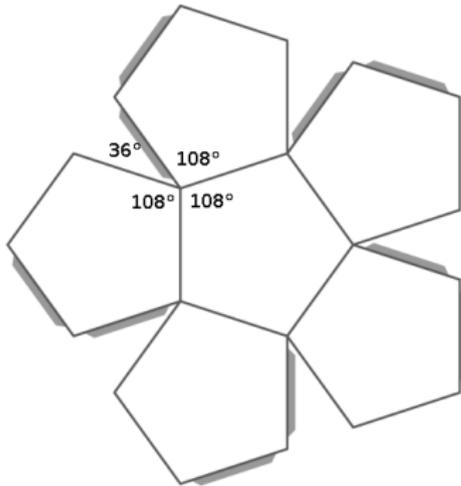
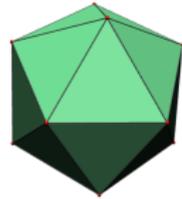
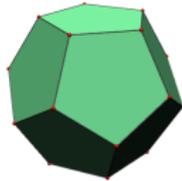
$$\subset \mathbb{R}^2$$



Platonische Körper



Bastelanleitung



$$\mathbb{R}^4 \longleftrightarrow \mathbb{R}^3$$



$$\mathbb{R}^4 \longleftrightarrow \mathbb{R}^3$$



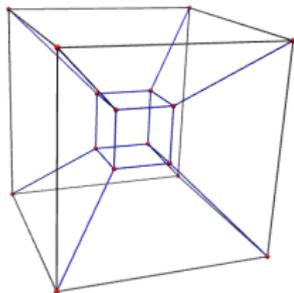
$$\mathbb{R}^4 \supset$$



$$\mathbb{R}^4 \longleftrightarrow \mathbb{R}^3$$

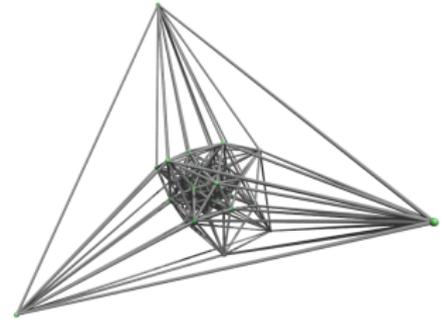
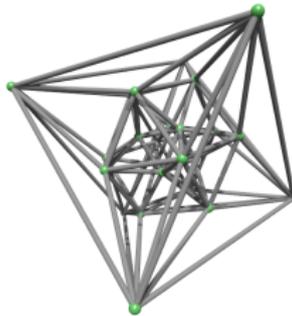
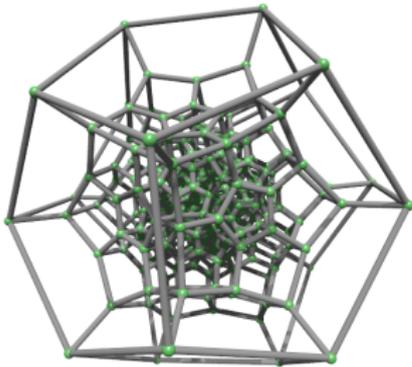
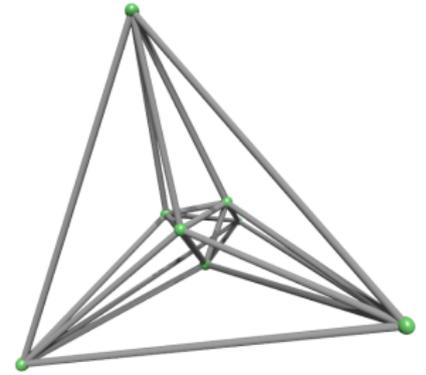
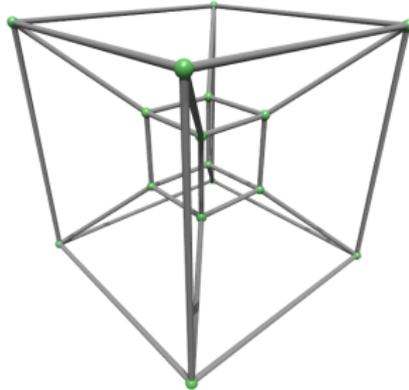
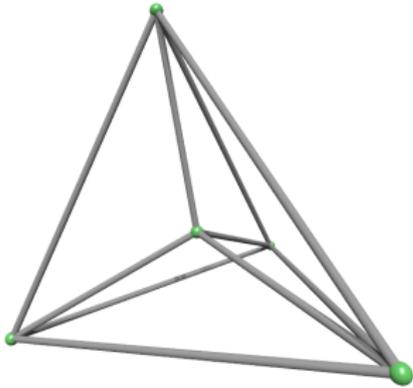


$$\mathbb{R}^4 \supset$$

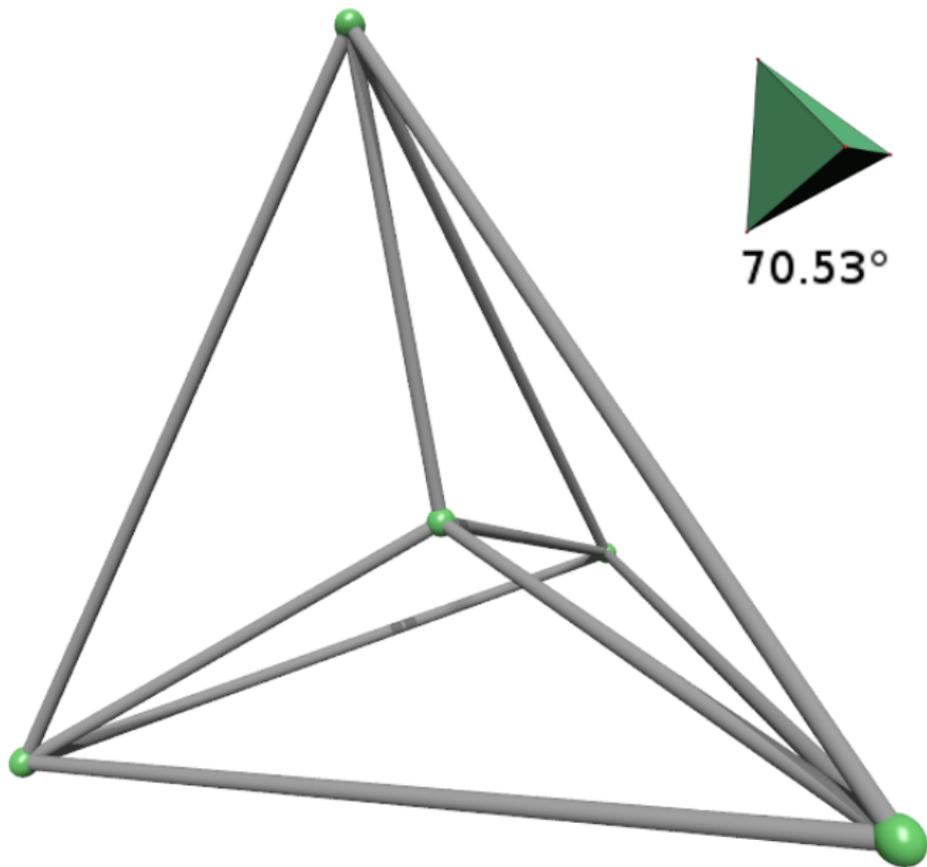


$$\subset \mathbb{R}^3$$

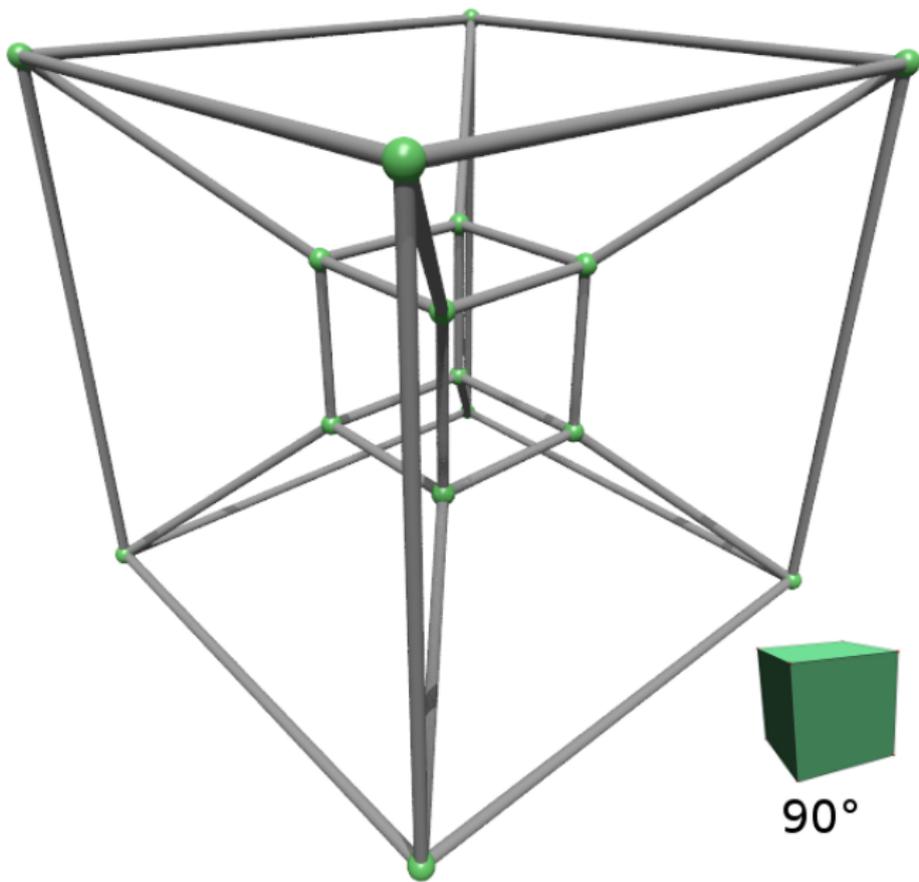
Hyperplatonische Körper



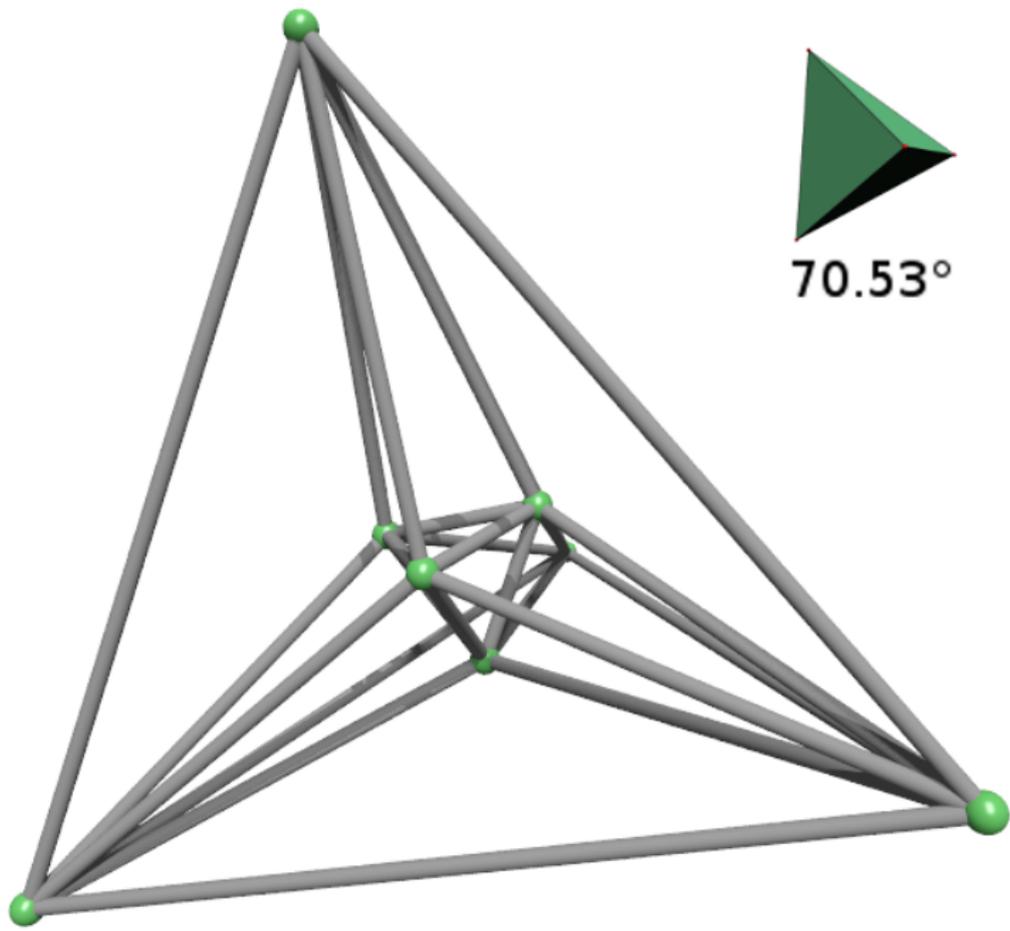
Hyper-Simplex



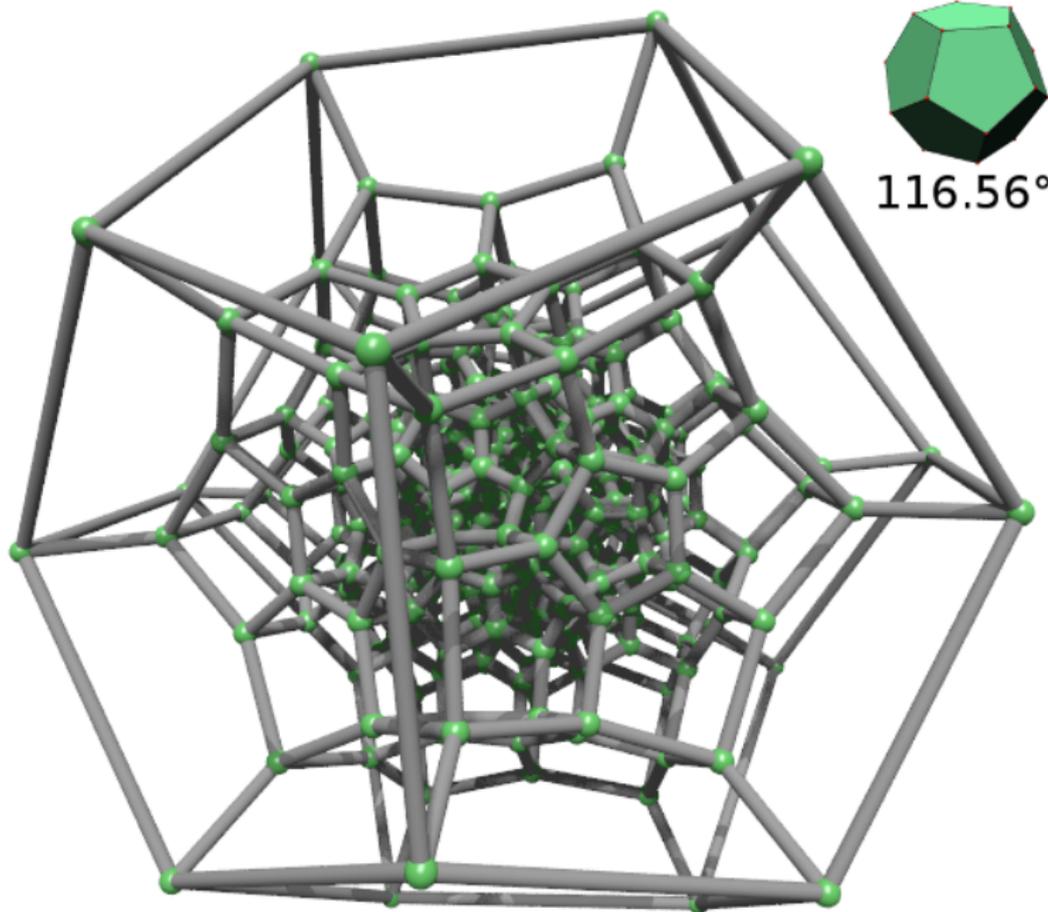
Hyper-Würfel



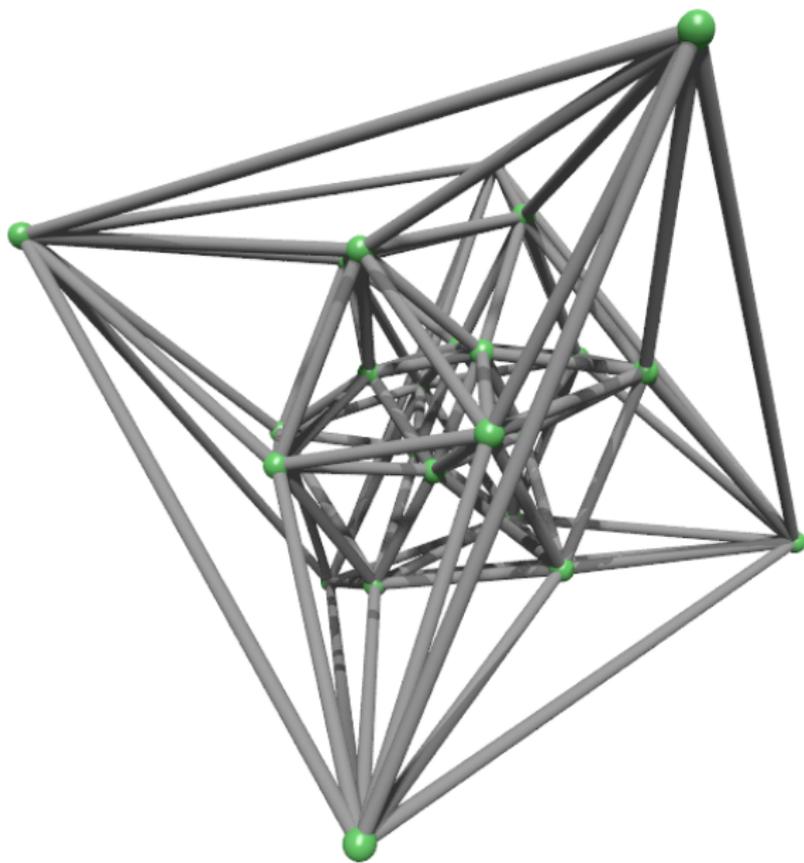
Hyper-Oktaeder



Hyper-Dodekaeder

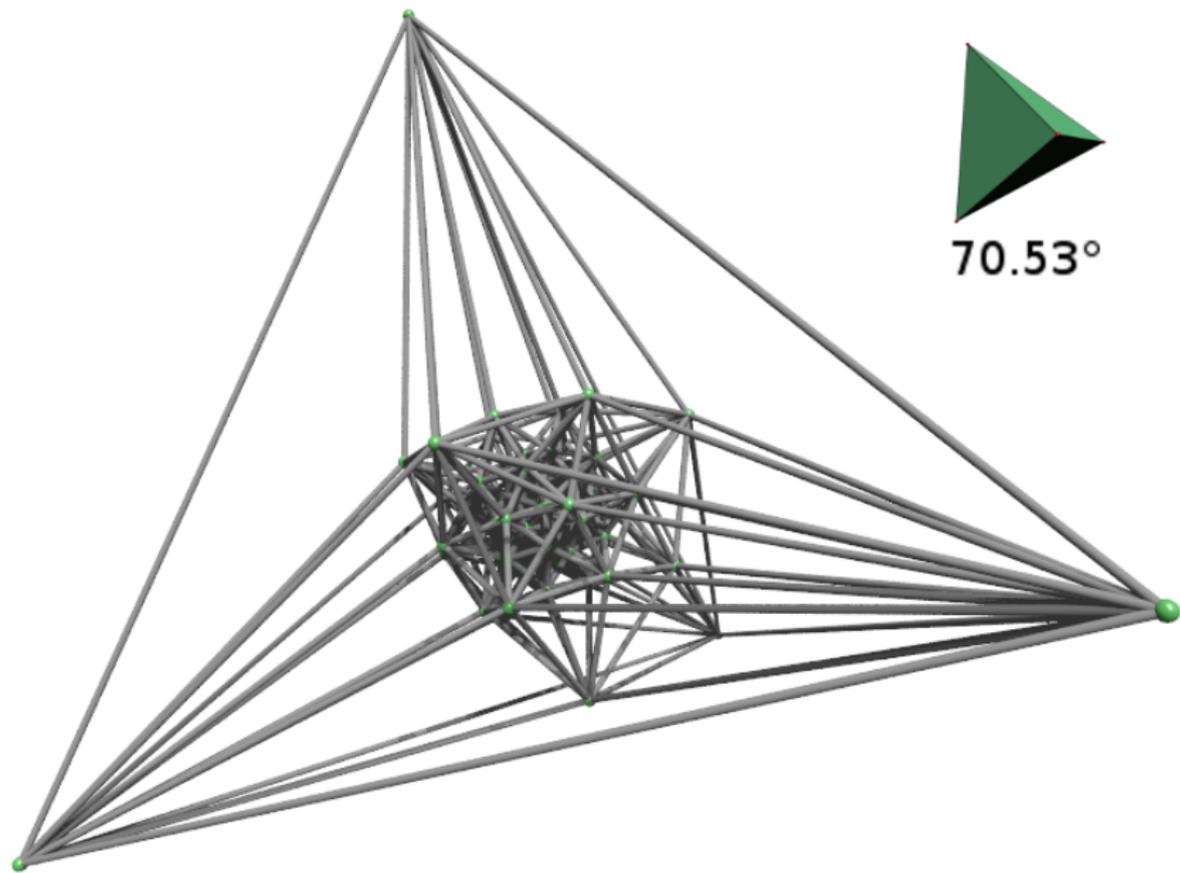


24-Zell

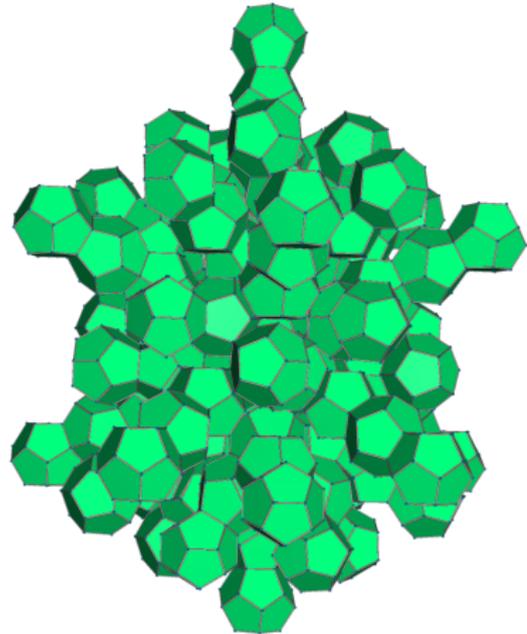
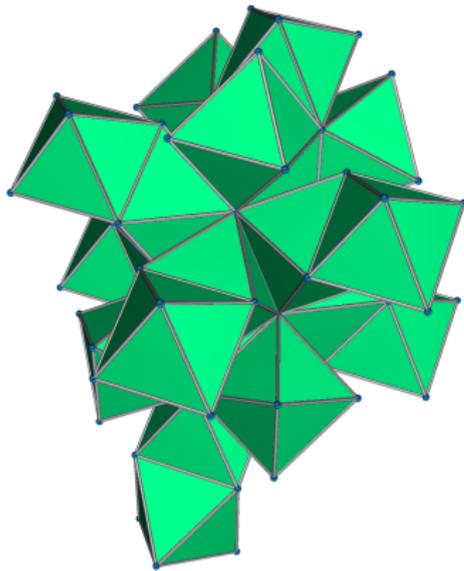
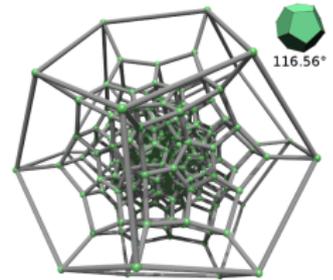
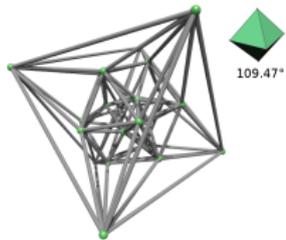


109.47°

Hyper-Icosaeder



Bastelanleitung

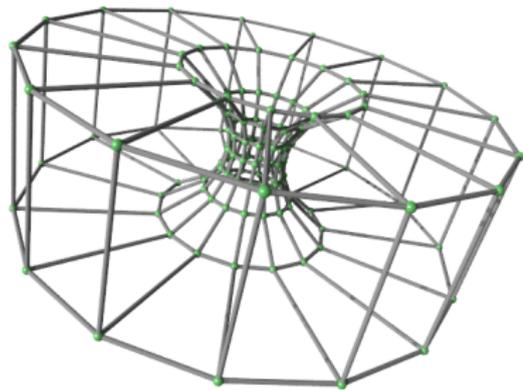


Anzahl von platonischen Körpern in \mathbb{R}^n

n	Anzahl
2	∞
3	5
4	6
5	3
6	3
7	3
\vdots	\vdots

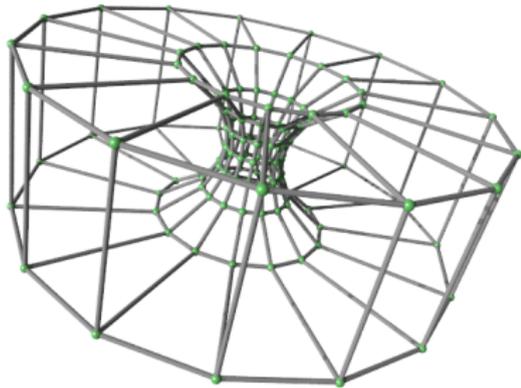
Ein Existenzbeweis

15-gon \times 10-gon

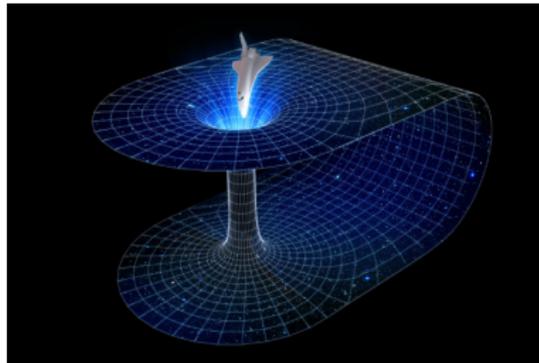


Ein Existenzbeweis

15-gon \times 10-gon

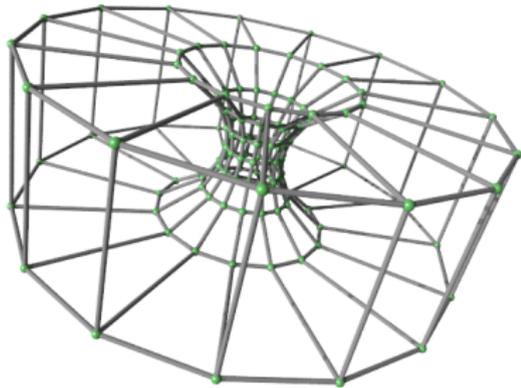


Wormloch

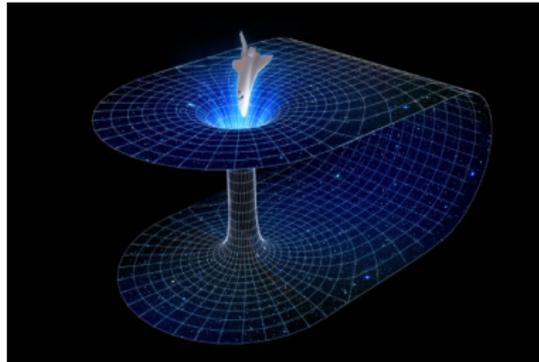


Ein Existenzbeweis

15-gon \times 10-gon



Wurmloch



Wurmlöcher existieren, q.e.d.

Danke für Ihre Aufmerksamkeit!