Exercise Sheet 10

Problem 1. Show that if $F \subset M \subset E$ are extensions of ∂-fields such that E/F and M/F are Picard-Vessiot extensions, then $\mathrm{Gal}^{\partial}(E/M) \leq \mathrm{Gal}^{\partial}(E/F)$ is a normal subgroup.

Problem 2. Let E/F be a Picard-Vessiot extension with ∂-Galois group G and let G^0 be the connected component of the identity of G.

1. Show that E^{G^0}/F is a finite Galois extension. (Use, without proof, the basic fact from algebraic geometry, that any affine variety has finitely many connected components).
2. Show that E/E^{G^0} is purely transcendental: there are no elements in $E \setminus E^{G^0}$ that are algebraic over E^{G^0}.