Estimation of High-Dimensional Low-Rank Matrices

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Suppose that we observe entries or, more generally, linear combinations of entries of an unknown $m \times T$-matrix A corrupted by noise. We are particularly interested in the high-dimensional setting where the number mT of unknown entries can be much larger than the sample size N. Motivated by several applications, we consider estimation of matrix A under the assumption that it has small rank. This can be viewed as dimension reduction or sparsity assumption. In order to shrink towards a low-rank representation, we investigate penalized least squares estimators with a Schatten-p quasi-norm penalty term, $p \leq 1$. We study these estimators under two possible assumptions – a modified version of the restricted isometry condition and a uniform bound on the ratio "empirical norm induced by the sampling operator/Frobenius norm". The main results are stated as non-asymptotic upper bounds on the prediction risk and on the Schatten-q risk of the estimators, where $q \in [p, 2]$. The rates that we obtain for the prediction risk are of the form rm/N (for $m = T$), up to logarithmic factors, where r is the rank of A. The particular examples of multi-task learning and matrix completion are worked out in detail. The proofs are based on tools from the theory of empirical processes. As a by-product we derive bounds for the kth entropy numbers of the quasi-convex Schatten class embeddings $S_p^M \hookrightarrow S_2^M$, $p < 1$, which are of independent interest. The talk is based on a joint work with Sasha Tsybakov.