Accuracy of Empirical Projections of High-Dimensional Gaussian Matrices

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Let $\varepsilon \in \mathbb{R}^{M \times M}$ be a centered Gaussian matrix whose entries are independent with variance $\sigma^2$. For $X = C + \varepsilon$ with a deterministic matrix $C$ and $\mathcal{S}_{M,r}$ denoting the set of all orthogonal projections onto $r$-dimensional subspaces of $\mathbb{R}^M$, the accuracy

$$E\|\tilde{\pi}_r X\|_{\mathcal{S}_2}^2 - \sup_{\tilde{\pi}_r \in \mathcal{S}_{M,r}} E\|\tilde{\pi}_r X\|_{\mathcal{S}_2}^2$$

of reduced-rank projections of $X$ is studied, where $\tilde{\pi}_r$ maximizes $\|\tilde{\pi}_r X\|_{\mathcal{S}_2}^2$ over $\mathcal{S}_{M,r}$. It is shown that a combination of amplitude and shape of the singular value spectrum of $C$ is responsible for the quality of the empirical reduced-rank projection, which we quantify for some prototype matrices $C$. Our approach does not involve analytic perturbation theory of linear operators and covers the situation of multiple singular values in particular. The main proof relies on a bound on the supremum over some non-centered process with Bernstein tails which is built on a slicing of the Grassmann manifold along a geometric grid of concentric Hilbert-Schmidt norm balls. The results are accompanied by lower bounds under various assumptions. Consequences on statistical estimation problems, in particular in the recent area of low-rank matrix recovery, are discussed.