Accuracy of Empirical Projections of High-Dimensional Gaussian Matrices

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Let $\varepsilon \in \mathbb{R}^{M\times M}$ be a centered Gaussian matrix whose entries are independent with variance σ^2. For $X = C + \varepsilon$ with a deterministic matrix C and $\mathcal{S}_{M,r}$ denoting the set of all orthogonal projections onto r-dimensional subspaces of $\mathbb{R}^M$, the accuracy

$$\mathbf{E}\|\widehat{\pi}_r X\|_{S_2}^2 - \sup_{\widetilde{\pi}_r \in \mathcal{S}_{M,r}} \mathbf{E}\|\widetilde{\pi}_r X\|_{S_2}^2$$

of reduced-rank projections of X is studied, where $\widehat{\pi}_r$ maximizes $\|\widetilde{\pi}_r X\|_{S_2}^2$ over $\mathcal{S}_{M,r}$. It is shown that a combination of amplitude and shape of the singular value spectrum of C is responsible for the quality of the empirical reduced-rank projection, which we quantify for some prototype matrices C. Our approach does not involve analytic perturbation theory of linear operators and covers the situation of multiple singular values in particular. The main proof relies on a bound on the supremum over some non-centered process with Bernstein tails which is built on a slicing of the Grassmann manifold along a geometric grid of concentric Hilbert-Schmidt norm balls. The results are accompanied by lower bounds under various assumptions. Consequences on statistical estimation problems, in particular in the recent area of low-rank matrix recovery, are discussed.