Phase transition in random walk distances on graphs

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We study the family of p-resistances on graphs. This family generalizes the standard resistance distance. We prove that for any fixed graph, for $p = 1$ the p-resistance coincides with the shortest path distance, for $p = 2$ it coincides with the standard resistance distance, and for p to infinity it converges to the inverse of the minimal s-t-cut in the graph. We consider the special case of random geometric graphs (such as k-nearest neighbor graphs) when the number n of vertices in the graph tends to infinity. We prove that an interesting phase transition takes place. There exist two critical thresholds p^* and p^{**} such that if $p < p^*$, then the p-resistance depends on meaningful global properties of the graph, whereas if $p > p^{**}$, it only depends on trivial local quantities and does not convey any useful information.