Multiple testing over a continuous domain of null hypotheses

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We introduce a theoretical framework for performing statistical hypothesis testing simultaneously over a fairly general, possibly uncountably infinite, set of null hypotheses. This extends the standard statistical setting for multiple hypotheses testing, which is restricted to a finite set. This work is motivated by numerous modern applications where the observed signal is modeled by a stochastic process over a continuum. As a measure of type I error, we extend the concept of false discovery rate (FDR) to this setting. Our main result shows how to use the p-value process to control the FDR at a nominal level, either under arbitrary dependence of p-values, or under the assumption that the finite dimensional distributions of the p-value process have positive correlations of a specific type (weak PRDS). The interest of this approach is demonstrated in several non-parametric examples.