

Contents

| | |
|---|-----------|
| Preface | vii |
| 1. The Basics | 1 |
| 1.1 Graphs* | 2 |
| 1.2 The degree of a vertex* | 5 |
| 1.3 Paths and cycles* | 6 |
| 1.4 Connectivity* | 10 |
| 1.5 Trees and forests* | 13 |
| 1.6 Bipartite graphs* | 17 |
| 1.7 Contraction and minors* | 18 |
| 1.8 Euler tours* | 21 |
| 1.9 Some linear algebra | 23 |
| 1.10 Other notions of graphs | 27 |
| Exercises | 29 |
| Notes | 34 |
| 2. Matching, Covering and Packing | 37 |
| 2.1 Matching in bipartite graphs* | 38 |
| 2.2 Matching in general graphs ^(*) | 43 |
| 2.3 The Erdős-Pósa theorem | 47 |
| 2.4 Tree packing and arboricity | 50 |
| 2.5 Path covers | 54 |
| Exercises | 56 |
| Notes | 59 |

* Sections marked by an asterisk are recommended for a first course.
Of sections marked ^(*), the beginning is recommended for a first course.

| | |
|--|-----|
| 3. Connectivity | 63 |
| 3.1 2-Connected graphs and subgraphs* | 63 |
| 3.2 The structure of 3-connected graphs ^(*) | 66 |
| 3.3 Menger's theorem* | 71 |
| 3.4 A -paths and Mader's theorem | 76 |
| 3.5 Linking pairs of vertices ^(*) | 79 |
| Exercises | 87 |
| Notes | 90 |
| 4. Planar Graphs | 93 |
| 4.1 Topological prerequisites* | 94 |
| 4.2 Plane graphs* | 96 |
| 4.3 Drawings | 102 |
| 4.4 Planar graphs: Kuratowski's theorem* | 106 |
| 4.5 Algebraic planarity criteria | 111 |
| 4.6 Plane duality | 114 |
| Exercises | 117 |
| Notes | 121 |
| 5. Colouring | 123 |
| 5.1 Colouring maps and planar graphs* | 124 |
| 5.2 Colouring vertices* | 126 |
| 5.3 Colouring edges* | 131 |
| 5.4 List colouring | 134 |
| 5.5 Perfect graphs | 139 |
| 5.6 χ -bounded graph properties | 146 |
| Exercises | 148 |
| Notes | 152 |
| 6. Flows | 155 |
| 6.1 Circulations ^(*) | 156 |
| 6.2 Flows in networks* | 158 |
| 6.3 Group-valued flows | 161 |
| 6.4 k -Flows for small k | 165 |
| 6.5 Flow-colouring duality | 167 |
| 6.6 Tutte's flow conjecture | 171 |
| Exercises | 174 |
| Notes | 176 |

| | |
|---|-----|
| 7. Extremal Graph Theory | 179 |
| 7.1 Subgraphs* | 180 |
| 7.2 Minors ^(*) | 185 |
| 7.3 Hadwiger's conjecture* | 189 |
| 7.4 Szemerédi's regularity lemma | 192 |
| 7.5 Applying the regularity lemma | 200 |
| 7.6 Two regularity tools | 206 |
| 7.7 Szemerédi's theorem | 216 |
| Exercises | 219 |
| Notes | 223 |
| 8. Infinite Graphs | 227 |
| 8.1 Basic notions, facts and techniques* | 228 |
| 8.2 Paths, trees, and ends ^(*) | 237 |
| 8.3 Homogeneous and universal graphs* | 246 |
| 8.4 Connectivity and matching | 249 |
| 8.5 Recursive structures | 260 |
| 8.6 Graphs with ends: the complete picture | 263 |
| 8.7 The topological cycle space | 272 |
| 8.8 Infinite graphs as limits of finite ones | 277 |
| Exercises | 280 |
| Notes | 293 |
| 9. Ramsey Theory for Graphs | 303 |
| 9.1 Ramsey's original theorems* | 304 |
| 9.2 Ramsey numbers ^(*) | 307 |
| 9.3 Induced Ramsey theorems | 310 |
| 9.4 Ramsey properties and connectivity ^(*) | 316 |
| Exercises | 319 |
| Notes | 320 |
| 10. Hamilton Cycles | 323 |
| 10.1 Sufficient conditions* | 323 |
| 10.2 Hamilton cycles and degree sequences | 327 |
| 10.3 Hamilton cycles in the square of a graph | 330 |
| Exercises | 335 |
| Notes | 336 |

| | |
|---|-----|
| 11. Random Graphs | 339 |
| 11.1 The notion of a random graph* | 340 |
| 11.2 The probabilistic method* | 345 |
| 11.3 Properties of almost all graphs* | 348 |
| 11.4 Threshold functions and second moments | 351 |
| Exercises | 358 |
| Notes | 360 |
| 12. Graph Minors | 363 |
| 12.1 Well-quasi-ordering ^(*) | 364 |
| 12.2 The graph minor theorem for trees | 365 |
| 12.3 Tree-decompositions ^(*) | 367 |
| 12.4 Tree-width ^(*) | 371 |
| 12.5 Tangles | 376 |
| 12.6 Tree-decompositions and forbidden minors | 391 |
| 12.7 The graph minor theorem ^(*) | 396 |
| Exercises | 405 |
| Notes | 412 |
| A. Infinite sets | 417 |
| B. Surfaces | 423 |
| Hints for all the exercises | 431 |
| Index | 433 |
| Symbol index | 451 |