# HODGE THEORY OF TORIC CALABI-YAU DEGENERATIONS

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## Small seminar series

I plan to give a seminar (4 sessions of 90minutes) on the Hodge-theoretic aspects of the Gross-Siebert program on toric Calabi-Yau degenerations and their canonical smoothings. We descend from the abstract theory to explicit computations of Hodge numbers by tropical data. I will present the main facts without digging into technical details too much and concentrate on illustrative examples.

**Session 1-2.** I intend to give an overview and a general introduction into the theory of variations and degenerations of Hodge structures as laid out in [Griffith],[Schmid] and [Steenb1], [Steenb2]. In particular, we will define mixed Hodge structures [Del2],[Del3], write them down explicitly for normal crossing varieties and understand how these compare to the mixed Hodge structures on the total space of a normal crossing degeneration with maximal unipotent monodromy. If there is time we will discuss the canonical affine structure on the Calabi-Yau moduli space at maximal unipotent boundary points.

**Session 3-4.** We review Gross and Siebert's computation for the Hodge groups of a toric Calabi-Yau degeneration [GrSi1],[GrSi2] whose central fibre is a special boundary point of the Calabi-Yau moduli space. We introduce the affine Hodge groups which can be computed from purely tropical data by means of a Čech-complex. We describe the algorithm to compute these for a degeneration constructed from a reflexive polytope. I will demonstrate some examples by means of a GAP toolbox for the subdivisions of reflexive polytopes and affine hodge number calculation which I have developed in 2006. We discuss situations where the affine Hodge numbers differ from the Hodge numbers of a smooth Calabi-Yau fibre in the family and explain the differences with the work in [Rud]. In particular, we show how to compute the additional contributions and how these techniques can be used to work with degenerations which feature only relatively few irreducible components in the special fibre improving computability.

## References

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