Universität Hamburg
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Fachbereich Mathematik
Analysis und Differentialgeometrie

Prof. Dr. Vicente Cortés

MSc Lecture Course

Analytic Methods in Affine Differential Geometry

(Vicente Cortés, Winter Semester 2012/13)

The main topic will be the geometry of hypersurfaces in affine space, in which a parallel volume form has been fixed. By an idea due to Blaschke, this allows to define a canonical transversal vector field on any oriented nondegenerate hypersurface, which is called the *affine normal*. It is obtained by solving a system of geometric differential equations, which involve the ambient volume form. There is an interesting geometry associated with the affine normal, including a pseudo-Riemannian metric and a connection on the hypersurface. The metric, which is called the *Blaschke metric*, is definite precisely when the hypersurface is locally convex. We will be especially interested in that case. A particularly interesting class of hypersurfaces are those for which the affine normal lines meet in one point or at infinity. These are called proper and improper *affine hyperspheres*, respectively. Affine hyperspheres are described by solutions to nonlinear partial differential equations of Monge-Ampère type. Proper affine hyperspheres fall into two classes: elliptic and the hyperbolic affine hyperspheres. The course will lead to highlights of the theory, such as Cheng and Yau's theorem on the completeness of affine hyperspheres, to Calabi's theorem on complete elliptic affine hyperspheres and to the Calabi-Pogorelov theorem on complete improper affine hyperspheres. The proofs are based on a fascinating interplay of geometric and analytic arguments, which will be carefully developed.

First reading: K. Nomizu and T. Sasaki, Affine Differential Geometry, Cambridge Tracts in Mathematics 111, Cambridge University Press, Cambridge, 1994.
Lecture Course: Monday and Thursday, 10:15-11:45, H6
Tutorials: Monday, 12:15-13:45, Geom 430