Open problems about infinite matroids - day IV

4.1 Let M, N and L be three matroids that have the same finite circuits and finite cocircuits but where M is finitary and N is cofinitary. Is it true that every finite minor of L is also a minor of M?

Motivation: M and N have the same finite minors and L sits in some sense between them. In particular, if M and N are binary, is then also L binary?

4.2 Let G be an arbitrary graph and M a tame matroid such that the finite circuits of M are the edge sets of finite cycles of G and the finite cocircuits of M are the finite bonds of G. What can you say about the structure of M? More precisely, is M determined by the set of ends used by the infinite circuits of M together with the set of those pairs (v, ω) consisting of a vertex v and an end ω such that some ray from v to ω is a circuit of M?

Motivation: It is a natural question to determine those matroids associated to a graph.

4.3 Let (T, M) be a nice tree of matroids such that all matroids $M(t)$ are finite and binary. Let N be a tame matroid such that its finite circuits are the $\emptyset$-circuits and its finite cocircuits are the $\emptyset$-cocircuits. What can you say about the structure of N? More precisely, is N always a Ψ-matroid for (T, M)?

Motivation: This would give a better understanding of the possible matroids that could be built from trees of matroids.

4.4 Is there a tame (binary) matroid that cannot be constructed from a tree of matroids over finitary and cofinitary matroids (along finite order separations)?

Motivation: So far all known tame matroids can be constructed from such a tree of matroids.

4.5 (Wagner's Conjecture for countable graphs) Is the class of countable graphs well-quasi ordered?

Motivation: True for finite graphs but false for uncountable graphs.

4.6 If a presentation Π is neat with respect to $\mathcal{F}$, must it then also be stellar with respect to $\mathcal{F}$? Motivation: If true, then the tree of matroids construction would be more flexible.