Open problems about infinite matroids - day I

1.1. Let $\mathcal{C}$ and $\mathcal{D}$ be sets of subsets of a set E satisfying (O1), (O2), (O3) and (O3*). Is there a matroid M with ground set E such that $\mathcal{C}(M) \subseteq \mathcal{C} \subseteq \mathcal{S}(M)$ and $\mathcal{C}(M^*) \subseteq \mathcal{D} \subseteq \mathcal{S}(M^*)$? What if we also require $\mathcal{C}$ and $\mathcal{D}$ to satisfy (T)? Motivation: This is true if E is countable.

1.2. Is every finitary matroid also k-nearly finitary for some k? Motivation: similar to Halin's Theorem.

1.3. Is the class of nearly finitary matroids the largest class of matroids including all finitary matroids and closed under taking unions of matroids? Motivation: it is closed under these operations, and almost any other attempt to take a union of 2 matroids seems not to give a matroid.

1.4 Let M_1 and M_2 be matroids on the same ground set E. Must there exist a matroid M on a larger ground set $E \dot\cup P \dot\cup Q$ such that $M_1 = M/P\backslash Q$ and $M_2 = M\backslash P/Q$? Motivation: To be revealed later