Infinite matroid theory exercise sheet 13

1.* Let M be a finitary matroid which is linearly representable over a field k. Show that M is also thin sums representable over k.

2. Show that the class of tame thin sums matroids representable over a given field k is closed under duality and under taking minors.

3. Let M be a k-paintable matroid, with the usual notation for the functions on the circuits and cocircuits. Let B be a base of M. Show that for any circuit C of M we have

$$c_C = \sum_{e \in C \setminus B} \frac{c_C(e)}{c_{C_e^B}(e)} c_{C_e^B} .$$

4. Let M be a matroid such that the empty set is not a base of M. Show that the set of sets of the form $B - e$, where B is a base of M and $e \in B$, is the set of bases of a matroid.

5. Give an example of a tame matroid M which is not thinly representable over $\mathbb{Q}$, but all of whose finite minors are representable over $\mathbb{Q}$.