## Exercises - day IV

1. Prove that the union of two finitary matroids is a matroid.

2. Let $G$ be a graph with two vertices $x, y \in V(G)$ and let $F \subseteq E(G)$. Show that if every finite cut separating $x$ from $y$ meets $F$, then the closure of $F$ in $|G|$ contains an $x$-$y$-arc. What happens in $|G|_\Psi$?

3. Let $M$ be a connected infinite matroid such that all its cocircuits are finite. Let $e$ be an element of the ground set $E$ of $M$. Using a compactness argument, prove that there is a subset $C$ of $E$ containing $e$ and such that $C$ does not include any finite circuit of $M$ or meet any cocircuit of $M$ just once. Deduce that $M$ has an infinite circuit. Show that any matroid all of whose circuits and cocircuits are finite must be a direct sum of finite matroids.

   **Reminder:**

   Let $\mathcal{I}$ and $\mathcal{J}$ be sets of subsets of a set $E$. We define the *union* of $\mathcal{I}$ and $\mathcal{J}$ as $\mathcal{I} \vee \mathcal{J} = \{I \cup J | I \in \mathcal{I}, J \in \mathcal{J}\}$.