Exercises - day II

1. Prove that there are $2^{2^{\aleph_0}}$ non-isomorphic matroids on a countable ground set.

2. Let $\mathcal{I}$ and $\mathcal{J}$ be sets of subsets of a set E. We let $\mathcal{I} \vee \mathcal{J} = \{I \cup J | I \in \mathcal{I}, J \in \mathcal{J}\}$. We say that $\mathcal{I}$ is *finitary* if an infinite set is in $\mathcal{I}$ if and only if all its finite subsets are. We say that $\mathcal{I}$ is *downclosed* if $I \in \mathcal{I}$, then all subsets of I are also in $\mathcal{I}$.

   Given two finitary downclosed sets $\mathcal{I}$ and $\mathcal{J}$, prove that $\mathcal{I} \vee \mathcal{J}$ is finitary.

3. In order to answer the following question you may assume that every 3-connected tame matroid has a $U_{2,4}$- or $M(K_4)$-minor.

   A finite matroid is *series-parallel* if it can be recursively constructed from adding parallel or serial edges. It is well-known that a finite connected matroid is series-parallel if and only if it does not have a $U_{2,4}$- or $M(K_4)$-minor.

   Find the right extension of the above theorem to infinite matroids.