

INTRODUCTION TO HIGGS BUNDLES BLOCK COURSE, SUMMER TERM 2018

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This is an advanced graduate course for master and phd students as well as researchers who are interested in Higgs bundles. In particular the course will serve as a preparation for the GRK summer school on Higgs bundles.

PREREQUISITES

A good working knowledge of complex geometry as well as familiarity with algebraic geometry are required.

DESCRIPTION

Higgs bundles were introduced by Nigel Hitchin in 1987 while studying the dimensional reduction of the self-duality equations from four to two dimensions. A Higgs bundle on a Riemann surface is a pair (\mathcal{E}, Φ) , where \mathcal{E} is a holomorphic vector bundle and Φ , the Higgs field, is a holomorphic 1-form with values in the bundle of endomorphisms of \mathcal{E} . It turned out that Higgs bundles on Riemann surfaces and their moduli spaces have a rich mathematical structure. They are important for a number of research areas such as gauge theory, Kähler and hyperkähler geometry, integrable systems as well as mirror symmetry and Langlands duality. The goal of the course will be to offer an introduction to the basic concepts involved in the study of Higgs bundles.

TOPICS OF THE COURSE

- (1) Review of complex and Kähler geometry
- (2) Hyperkähler manifolds
- (3) Moduli of bundles
- (4) Higgs bundles

CREDIT REQUIREMENTS

The course can give 3 credit points for master students. The requirements are active participation in the lectures as well as an oral exam.

LOGISTICS

The lectures will take place in Geomatikum in the period from September 4th until September 7th, times and location will be determined later. The course has a home-page with updates: <http://www.math.uni-hamburg.de/home/alim/higgsbundles.html> There will furthermore be a mailing list for updates on the course, please send an email to murad.alim@uni-hamburg.de if you want to receive these emails.

LITERATURE

A good introductory reference are the lecture notes of Andrew Neitzke which can be found at: <https://www.ma.utexas.edu/users/neitzke/teaching/392C-higgs-bundles/higgs-bundles.pdf>

Further background material for the course can be found in [GH78], [Mor07], [Wel08].

REFERENCES

- [GH78] Phillip Griffiths and Joseph Harris. *Principles of algebraic geometry*. Wiley-Interscience [John Wiley & Sons], New York, 1978. Pure and Applied Mathematics.
- [Mor07] Andrei Moroianu. *Lectures on Kähler geometry*, volume 69 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 2007.
- [Wel08] Raymond O. Wells, Jr. *Differential analysis on complex manifolds*, volume 65 of *Graduate Texts in Mathematics*. Springer, New York, third edition, 2008. With a new appendix by Oscar Garcia-Prada.

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