

Degenerate U -statistics under weak dependence: Asymptotics, bootstrap and applications in statistics

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Numerous test statistics can be approximated by statistics of U - or V -type. In the case of i.i.d. random variables the limit distributions can be derived by a spectral decomposition of the kernel if the latter is square integrable. To use the same method of proof in the context of dependent data often requires restrictive assumptions on the associated eigenfunctions. However, their validity is quite difficult or even impossible to verify for many concrete examples in statistical hypothesis testing. In this talk, we devise new approaches to the asymptotic distributions of degenerate U - and V -statistics for weakly dependent random variables. We avoid any high-level assumption that can hardly be checked in applications. Instead only some moment constraints and smoothness assumptions concerning the kernel are required.

The limit distributions of U - and V -statistics for both independent and weakly dependent observations cannot be used directly since they depend on certain parameters which in turn depend on the underlying situation in a complicated way. Therefore, problems arise as soon as critical values for test statistics of U - and V -type have to be determined. The bootstrap offers a convenient way to circumvent these problems. There are already various papers on the validity of different bootstrap methods for degenerate U -statistics of i.i.d. data. Here, we derive the consistency of general bootstrap methods for those statistics in the case of weakly dependent observations.

Finally, we apply our results to construct bootstrap-based goodness-of-fit tests of L_2 -type for time series.