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Fakultät für Mathematik, Informatik
und Naturwissenschaften
Fachbereich Mathematik

Lothar-Collatz-Kolloquium für Angewandte Mathematik

Donnerstag, den 26. Juni 2025, um 17:15 Uhr, im Hörsaal 5

Prof. Dr. Christian Kreuzer*
(Technische Universität Dortmund, Fakultät für Mathematik, LS X)

Pressure robust discretisations of the nonlinear Stokes equations

Zusammenfassung/Abstract:

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Pressure robust discretisations of the nonlinear Stokes equations

Christian Kreuzer

Abstract

For standard conforming mixed discretisations of the power law Stokes problem

$$\begin{aligned} -\operatorname{div}|\nabla u|^{r-2}\nabla u + \nabla p &= f \\ -\operatorname{div} u &= 0, \end{aligned}$$

suboptimal convergence rates are observed in the shear thickening case (i.e. for exponents $r > 2$) theoretically and in simulations. The reason for this lies in some missmatch in the interaction of the velocity and the pressure error: The pressure p enters as the linear Lagrange multiplier according to the solenoidality condition, while the fluid velocity u enters in a nonlinear fashion.

A close by approach to overcome this drawback, is to use exactly divergence free finite element spaces, which fully decouples the velocity from the pressure approximation. This, however, comes with a number of other disadvantages, like conditions on the polynomial degree and the mesh geometry.

Another option is to use pressure robust methods: In this talk, we shall demonstrate for the Stokes element consisting of Crouzeix-Raviart first order velocities and piecewise constant pressure, that pressure robust techniques can be used to remedy the suboptimal approximation results for nonlinear Stokes problems. In particular, we prove regularity free (almost) quasi optimality of the method. From this we conclude optimal order convergence of the velocity error in the quasi-norm independent of the pressure.

This is a joint work with Lars Diening, Adrian Hirn, and Pietro Zanotti