Universität Hamburg
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Fakultät für Mathematik, Informatik
und Naturwissenschaften
Fachbereich Mathematik

Lothar-Collatz-Kolloquium für Angewandte Mathematik

Donnerstag, den 15. Juni 2023, um 17:15 Uhr, im Hörsaal 5

Prof. Dr. Philipp Birken*
(Lund University, Centre for Mathematical Sciences)

"Iterative solvers versus structure preserving discretizations in compressible CFD"

Zusammenfassung/Abstract:

We consider computational fluid dynamics for compressible flows. This plays an important role in the design of air planes, new renewable jet fuels, or in simulation of the atmosphere within weather and climate predictions. During the past decade, significant effort has been put into high order discontinuous Galerkin methods for these problems. A successful design principle has been to find methods that preserve properties of the differential equation, such as the underlying conservation law or the second law of thermodynamics in the form of an entropy inequality.

Here, we focus on implicit discretizations in time, in particular within entropy preserving space-time discretizations. To solve the arising nonlinear equation systems, iterative methods are needed. The leading question in this talk is these approximate solutions in turn preserve the properties of the discretization.

Firstly, we consider global and local conservation. As it turns out, global conservation, which corresponds to a linear invariant, is not difficult to achieve. Further, many commonly used methods preserve even the local conservation of an underlying implicit scheme. This includes pseudo-time iterations, Newton's method and Krylov subspace methods. However, there are prominent exceptions, in particular the Jacobi and Gauss-Seidel iterations. We present extensions of the Lax-Wendroff theorem for a fixed, finite number of iterations each time step. The iterative method defines a numerical flux that is inconsistent in general. We can describe the specific inconsistency as a form of slowed down time for pseudo-time iterations, Krylov methods, and thereby also Newton-Krylov methods. A simple technique based on the explicit Euler method can alleviate flux-inconsistency and can additionally be used to generate improved initial guesses for iterative methods.

Secondly, we look at entropy preservation of iterative methods, respectively of nonlinear invariants. As it turns out, standard iterative methods do not preserve these. We present various ways that Newton's method can be adjusted to fix this problem. Numerical methods illustrate the advantages and disadvantages of the various fixes.

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