Lothar-Collatz-Kolloquium für Angewandte Mathematik

Donnerstag, den 24. November 2022, um 17:15 Uhr, im Hörsaal 5

Prof. Dr. Robert Scheichl
(Ruprecht-Karls-Universität Heidelberg, Institut für Angewandte Mathematik)

“Efficient Rare Event Estimation in High Dimensions”

Zusammenfassung/Abstract:

In this talk I will present two promising approaches for estimating rare event probabilities in high-dimensional PDE-constrained problems. The first approach is an adaptive multilevel version of subset simulation for complex physical systems, where the rare event probability is expressed as a product of conditional probabilities based on a sequence of nested failure domains of increasing size. By using different model resolutions and varying numbers of samples across the hierarchy of nested failure sets, the new estimator allows to dramatically reduce the computational cost. A key idea in our new estimator is the use of a posteriori error estimation combined with a selective mesh refinement strategy to guarantee the critical subset property that may be violated when changing model resolution from one failure set to the next. For a model problem from subsurface flow, the new multilevel estimator achieves efficiency gains of more than a factor 200 over standard subset simulation. The improved complexity can also be shown theoretically. An alternative approach for estimating rare event probabilities in high-dimensional problems is importance sampling. In recent work, we propose to approximate the optimal importance distribution in a general importance sampling problem as the pushforward of a reference distribution under a composition of order-preserving transformations. Each of the transformations is formed by a squared tensor-train (TT) decomposition, which together with a sequence of bridging densities provides a scalable ansatz to build order-preserving high-dimensional transformations even for concentrated density functions. To compute expectations over unnormalized probability distributions, we design a ratio estimator that estimates the normalizing constant using a separate importance distribution in TT format. This offers better theoretical variance reduction compared with self-normalized importance sampling, and thus opens the door for the efficient computation of rare event probabilities in Bayesian inference problems. Numerical experiments on problems constrained by differential equations show little to no increase in the computational complexity with the event probability going to zero, and allow to compute hitherto unattainable estimates of rare event probabilities for complex, high-dimensional posterior densities.

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