Name(s):

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Partial Differential Equations

Winter Semester 2017–2018

Worksheet 9 — Tuesday, December 12, 2017

We assume all the functions to be smooth, unless stated otherwise.

Problem 1

Consider the polar coordinates $\Psi : (r, \varphi, \theta) \mapsto (r\cos(\varphi)\sin(\theta), r\sin(\varphi)\sin(\theta), r\cos(\theta))$ on $\mathbb{R}^3$, where $r > 0, \varphi \in (-\pi, \pi)$ and $\theta \in (0, \pi)$. Transform the Laplacian in polar coordinates, i.e. find a differential operator L of second order such that $\Delta u = L(u \circ \Psi)$ for all $u \in C^2(\mathbb{R}^3)$.

Problem 2

(a) Write down the characteristic equations for the PDE

$$u_t + b.\mathcal{D}u = f \quad \text{in } \mathbb{R}^n \times (0, \infty) \tag{1}$$

where $b \in \mathbb{R}^n$, $f = f(x,t)$.

(b) Use the characteristic ODE to solve (1) subject to the initial condition

$$u = g \quad \text{on } \mathbb{R}^n \times \{t = 0\}.$$

Makes sure that your answer agrees with the formula in the lecture.

Problem 3

Solve using characteristics:

(a)

$$x_1 u_{x_1} + x_2 u_{x_2} = 2u, \quad u(x_1, 1) = g(x_1).$$

(b)

$$u.u_{x_1} + u_{x_2} = 1, \quad u(x_1, x_1) = \frac{1}{2}x_1.$$

(c)

$$x_1.u_{x_1} + 2x_2.u_{x_2} = 3u, \quad u(x_1, x_2, 0) = g(x_1, x_2).$$

Problem 4

Show that the solution provided by Theorem 3.12 is unique, if there is only one p_0 such that $(p^0, g(x^0), x^0)$ admissible. Construct an example where uniqueness fails, if there exists more than one p^0 such that $(p^0, g(x^0), x^0)$ is admissible.

For December 19, 2017, at 08:15 A.M.