Name(s):

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Partial Differential Equations

Winter Semester 2017–2018

Worksheet 3 Tuesday, October 31, 2017

We assume all the functions to be smooth, unless stated otherwise.

Problem 1

Prove that there exists a constant C, depending only on n, such that

$$\max_{B(0,1)} |u| \leq C(\max_{\partial B(0,1)} |g| + \max_{B(0,1)} |f|)$$

whenever u is a smooth solution of

$$\begin{cases} -\Delta u = f & \text{in } x \in B^0(0,1), \text{ where } B^0(0,1) \text{ is the interior of } B(0,1) \\ u = g & \text{on } \partial B(0,1) \end{cases}$$

Hint: define $v(x) = u(x) + \frac{\max_{B(0,1)} |f|}{2n} . |x|^2$.

Problem 2

Assume that $u : \mathbb{R}^n \to \mathbb{R}$ is a harmonic function. Show that

(a) $\int_{\mathbb{R}^n} |u|^2 < \infty$ then $u = 0$.

(b) $\int_{\mathbb{R}^n} |\mathcal{D}u|^2 < \infty$ then $u = constant$.

Problem 3

Let $U \subset \mathbb{R}^n$ be a bounded open set. Consider a sequence u_n of harmonic functions such that u_n converges uniformly to u on U, when $n \to \infty$. Prove that u is a harmonic function.

Problem 4

Consider an increasing sequence $u_n : B(0,R) \to \mathbb{R}$ of harmonic functions, that is $u_n(x) \leq u_{n+1}(x)$ for every $x \in B(0,R)$. Assume that $u_n(0)$ is a Cauchy sequence. Prove that for every $r < R$, there exists a harmonic function u such that u_n converges uniformly to u in $B(0,r)$.

For November 07, 2017, at 08:15 A.M.