Name(s):

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Partial Differential Equations

Winter Semester 2017–2018

Worksheet 10 — Tuesday, December 19, 2017

Problem 1 Let X, Y be Banach spaces, let $V \subset X$ be a dense subspace of X and $A : V \to Y$ be a bounded linear operator. Show that there exists a unique extension $\tilde{A} : X \to Y$, i.e. a unique bounded linear operator $\tilde{A} : X \to Y$ such that $\tilde{A}|_V = A$.

Problem 2

As functions on $\mathbb{R}^n$ of the parameters α, β, determine for which $n \in \mathbb{N}$, $k \in \mathbb{N}$, the following functions belong to $H^k(B_{1/2}(0))$:

(a) $u(x) = |x|^\alpha$

(b) $u(x) = |\ln |x||^\beta$

(c) $u(x) = |x|^\alpha |\ln |x||^\beta$

Problem 3

Let U be a bounded open subset of $\mathbb{R}^n$ with smooth boundary. Prove that in general $H^1_0(U) \neq H^1(U)$, i.e. show that they are not equal for any U.

(Hint: consider the function equal to 1 on the unit ball of $\mathbb{R}^n$.)

Problem 4

Let $\Omega = B_1(0) \subset \mathbb{R}^2$ and $u : \Omega \to \mathbb{R}$ with

$$u(x) := \ln(|\ln \frac{1}{|x|}|).$$

Prove that $u \in H^1(\Omega) \backslash C(\Omega)$.

For January 09, 2018, at 08:15 A.M.