Exercises in Algebraic Topology (master)

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Summer term 2025

Exercise sheet no 9 due: 10th of June 2025, 13:45h in H3

1 (Products of Moore spaces) (4 + 1 points)

(1) Let A and B be arbitrary finitely generated abelian groups. Determine the homology groups of the product of the Moore spaces $M(A,n)$ and $M(B,m)$, $H_*(M(A,n) \times M(B,m))$, for arbitrary natural numbers $n, m \geqslant 1$.

(2) In the special case of $M(\mathbb{Z}/2\mathbb{Z}, 1) = \mathbb{R}P^2$, what is $H_*(\mathbb{R}P^2 \times \mathbb{R}P^2)$?

2 (Splittings) (3 + 2 points)

a) Give an explicit example for the fact that the splitting in the topological Künneth formula is not natural.

b) Let C_* be a free chain complex and let G be an abelian group. Show that the sequence

$$0 \to H_n(C_*) \otimes G \to H_n(C_* \otimes G) \to \mathrm{Tor}(H_{n-1}(C_*), G) \to 0$$

splits.

3 (Ext) (2 + 2 points)

For a free resolution $0 \longrightarrow R \xrightarrow{i} F \xrightarrow{p} A \longrightarrow 0$ and an abelian group B the cokernel of the map

$$\mathsf{Hom}(i, B) \colon \mathsf{Hom}(F, B) \to \mathsf{Hom}(R, B)$$

is $\mathsf{Ext}(A, B)$ and hence the sequence

$$0 \longrightarrow \mathsf{Hom}(A, B) \xrightarrow{\mathsf{Hom}(p,B)} \mathsf{Hom}(F, B) \xrightarrow{\mathsf{Hom}(i,B)} \mathsf{Hom}(R, B) \longrightarrow \mathsf{Ext}(A, B) \longrightarrow 0$$

is exact.

1) Determine $\mathsf{Ext}(A, B)$ if A is a finitely generated abelian group.

2) For natural numbers n and m give an explicit formula for $\mathsf{Ext}(\mathbb{Z}/n\mathbb{Z}, \mathbb{Z}/m\mathbb{Z})$.