Universität Hamburg
Janko Latschev Winter 2024/25

Symplectic Geometry

Problem Set 3

1. Consider the standard symplectic form $\omega = \sum_k dp_k \wedge dq_k$ on $\mathbb{R}^{2n}$. Find an explicit expression for the Poisson bracket $\{F, G\}$ of two functions F and G on this symplectic manifold.

2. On the symplectic manifold $(\mathbb{R}^4, \omega = dp_1 \wedge dq_1 + dp_2 \wedge dq_2)$ we consider the Hamiltonian system given by the Hamiltonian function

$$H(q,p) = \frac{p_1^2}{2} + \frac{p_2^2}{2} + e^{q_1 - q_2}.$$

a) Prove that this system is completely integrable by showing that the function $P(q,p) = p_1 + p_2$ is an integral of motion. What else needs to be checked?

b) For $c = (c_1, c_2) \in \mathbb{R}^2$ we consider the subset

$$M(c_1, c_2) := \{(q,p) \in \mathbb{R}^4 \,:\, H(q,p) = c_1, P(q,p) = c_2\} \subseteq \mathbb{R}^4.$$

Show that this subset is nonempty if and only if $4c_1 - c_2^2 > 0$.

c) Show that if the condition from part **b)** is satisfied, this subset is diffeomorphic to $\mathbb{R}^2$.

d) Conclude that this system has no periodic orbits.