Homework 6, due Monday 14 May

1. Prove in **HA**:

   (a) $x \cdot (y + z) = x \cdot y + x \cdot z$ [2 pts]

   (b) $\forall x \forall y (x = y \vee x \neq y)$ [2 pts]

   You may use that Robinson's axiom $x = 0 \vee \exists y (x = y + 1)$ is provable in **HA**, shown in class.

2. (a) Show that **HA** has the existence property: if **HA** $\vdash \exists x \varphi(x)$, then **HA** $\vdash \varphi(\overline{n})$ for some n. [2 pts]

   (b) Add a predicate $A(x)$ to the language of **HA** with the axiom $A(0) \wedge \forall x (A(x) \to A(x + 1))$, but do **not** add induction for formulas containing A. Show the disjunction property for this system. [2 pts]

   (c) Add a predicate $B(x)$ to the language of **HA** with the axiom $\exists x B(x)$. Does this system have the disjunction property? [2 pts]

3. Prove that the induction rule

$$\frac{\varphi(0), \forall x (\varphi(x) \to \varphi(x + 1))}{\forall x \varphi(x)}$$

   is equivalent to the induction schema of HA. [4 pts]

4. Show that **IQC** has the disjunction property (be careful: this exercise is not completely trivial!) [4 pts]