Homework 2, due Monday 14 February

1. Give Kripke counter-models to:

   (a) $\neg(p \wedge q) \to \neg p \vee \neg q$ [2pts]

   (b) $\neg(p \to q) \to p \wedge \neg q$ [2pts]

   (c) $[((p \to q) \to q) \wedge ((q \to p) \to p)] \to (p \vee q)$ [2pts]

2. Exercise 4 of the syllabus, on p 16:

   Prove that persistency transfers to formulas (i.e., if $w \models \phi$ and wRv then $v \models \phi$, for all propositional formulas ϕ). [4pts]

3. Show that $\Box p \to \Box\Box p$ characterizes the transitive frames. That is to say: give only the difficult direction of the proof, but give a non-constructive and a constructive proof, and discuss. [4 pts]

4.* Show that $\forall x A(x) \to \forall x B(x)$ and $\exists x \neg B(x) \to \exists x \neg A(x)$ are independent in intuitionistic predicate logic, by giving two models (it may be useful to note that the second formula is equivalent to $\forall x(\neg B(x) \to \exists y \neg A(y))$).

   [4 pts]