Wavelet density estimators on compact manifolds From concentration to adaptation

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I shall start with a review of recent results from harmonic analysis of Lie groups that allow for the construction of localised wavelet-, or needlet-, frames on compact manifolds due to Geller, Mayeli, Pesenson (2009, 2011). The idea is to suitably smooth the projector kernel on the span of the eigenfunctions of a Laplace-type operator on the manifold, which, if the manifold is homogeneous, gives rise to a Parseval frame with basis functions that have near-exponential localisation properties on the manifold. I shall then show how this can be employed to define the associated needlet density estimator based on an i.i.d. sample observed on the manifold. Combing standard concentration of measure arguments with Gaussian cubature formulae for the manifold one can prove fairly sharp concentration inequalities for the uniform fluctuations of the estimator. This is useful for various purposes, such as for instance testing for uniformity of the distribution of the sample, for the construction of sharp rate-adaptive density estimators or nonasymptotic confidence bands for the unknown density. Particular applications occur in astrophysics when the manifold equals the celestial sphere, but other examples such as d-dimensional unit spheres, projective spaces, Grassmann and Stiefel manifolds, also come to mind. This is joint work with G. Kerkyacharian and D. Picard.