Workshop on Nonlinear Dynamics

Symmetry, Networks, Attractors

In Anticipation of the 70th Birthday of Reiner Lauterbach

July 24 – July 26, 2024
Universität Hamburg

Welcome Address

This workshop highlights, recounts, and discusses historic and, most prominently, recent developments in the mathematical research subject of nonlinear dynamics and bifurcations with a particular focus on methods exploiting underlying structures. We are delighted to be able to bring together leading experts on different approaches to this field in order to establish and exploit links between them and pave the way for future progress. In particular, analyzing the interplay between symmetries, network structure, and potential attractors of a given system can lead to otherwise inaccessible insights.

On this occasion, we also anticipate Reiner Lauterbach's 70th birthday later this year and celebrate his contributions to the field of nonlinear dynamics.

Nonlinear processes are ubiquitous in nature, engineering, and sciences. An example that has recently been at the center of public attention is exponential disease spreading as the driver of the COVID-19 pandemic. The study of such processes, where a quantity of interest changes over time, relies on mathematical models which take the form of nonlinear dynamical systems. A general understanding of these systems poses various challenges. On one hand, systems arising in real-world applications are typically large and thus of very high dimension with a large number of degrees of freedom. Furthermore, frequently multiple processes on different spatial and temporal scales take place simultaneously. As a result, insight is oftentimes limited, restricted to small proxy examples, or relies heavily on numerical simulations. In particular, the enormous diversity of general dynamical systems requires a more fine-grained approach consisting of various different solution strategies. Subsequently, meaningful progress has been achieved and can be further expected for subclasses of systems with specific properties.

A very successful approach has been to consider dynamical systems that respect underlying structure. For example, symmetries of a physical object in consideration typically translate into symmetry properties of the equations that govern the dynamical process. This relation has been actively elucidated in the field of equivariant dynamics within the last 70 years. The key feature is that the symmetries of the equations of motion restrict and therefore organize possible dynamics, which offers insights into the problem at hand. Exploiting such structures has led to staggering results. Prominent examples include the classification of generic bifurcations, the discovery of symmetric chaos, and the unveiling of formerly unknown behavior in real world systems (such as convection rolls in Rayleigh-Bénard convection).

Collective dynamics of interacting individuals appear in countless applications such as neuroscience or robotics. Within the last 30, years such interaction networks have emerged as another structure that affects dynamics. In contrast to symmetries, network structure is not an intrinsic property of a system. For example, network structure in the equations of motion is typically not preserved under dimension reduction methods. This calls for adapted and entirely new techniques. Numerous important results have been developed. Examples include the exploitation of the inherently algebraic nature of networks to identify patterns of partial synchronization as well as the determination of critical agents and links in interconnected systems such as vulnerable points in power grids. Nonetheless, network dynamics are still a highly active area of research with many features still to be uncovered.

The concept of an attractor captures the long-term behavior of a dynamical system. Regular dynamics (equilibria, periodic orbits) can be distinguished from more complicated asymptotic regimes, which may be associated with e.g. heteroclinic or chaotic structures. Properties of a system, such as an inherent symmetry or network structure, significantly impact the possible attractors. Furthermore, restricting the analysis of a system to its attractor often reduces the dimension of the problem substantially.

To understand specific models that describe real world processes, one can now choose from very powerful and specialized results that came out of the impressive developments outlined above. On one hand, during the modelling process reasonable assumptions can lead to systems from one of those classes that are better understood. For example, the switching of the earth's magnetic field can be regarded as a heteroclinic process. On the other hand, application inspired systems often have structural properties like (approximate) symmetries (e.g., rotation symmetries of global climate models) or network structure (e.g., deep learning processes) or even both and can be addressed correspondingly. Finally, given model equations can, in certain scenarios, be brought into a form in which it exhibits underlying structures. For example, discretization even of unstructured and very complicated systems typically leads to reduced ones on a graph, a special case of a network dynamical system.

During the three days of the *Workshop on Nonlinear Dynamics*, leading experts, who have significantly shaped the research landscape and continue to do so, discuss current developments. It is a platform for sharing ideas and potentially triggering discoveries of yet unknown links or concepts. We thank all speakers, poster presenters, and everybody else joining the workshop for their active participation. The scientific presentations and off-programme discussions in combination with the purely social interactions are integral for making this event a truly exciting experience. Finally, we would like to express our gratitude to the Deutsche Forschungsgemeinschaft as well as to the members of the Department of Mathematics at the Universität Hamburg for their generous funding to make this workshop possible.

We are very excited to welcome all of you in Hamburg!

The organizing committee

Ingenuin Gasser	Sören von der Gracht
Maxim Kirsebom	Philipp Kunde
Alexander Lohse	Florian Noethen
Mara Sommerfeld	Jan Henrik Sylvester

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Abstracts

Sequences and branching of learned states in neural networks, with an application to decision making

24 Jul
9:15am

Pascal Chossat

CNRS - Inria Sophia Antipolis

It is a popular idea that concepts and learned items are stored in the associative memory as steady-states of a network of "units", where each unit is itself a neural mass (population of similar and highly interconnected neurons). Existence and stability is achieved by the learning process which shapes the synaptic connections between units. Retrieval of a learned item occurs when the system is placed in the basin of attraction of the corresponding steady-state. Learned states can lose stability through internal mechanisms such as synaptic short term depression (STD). The brain's processing of associations between learned concepts would then correspond to trajectories in the phase space, wandering between the corresponding states under the effect of STD combined with noise. In particular a sequence of related concepts would proceed as a chain of "jumps" from one state to the next, called latching dynamics, where each state shares active units with the next one (overlapping states). This idea has been sustained by numerical simulations of a "large" number of neurons under suitable hypotheses (see in particular [Lerner, I, Bentin, S and Shriki, O. 2012. "Spreading Activation in an Attractor Network With Latching Dynamics: Automatic Semantic Priming Revisited." *Cognitive Science*, 36: 1339-1382]). However a rigorous mathematical analysis of the phenomenon was lacking. Our model is aimed at filling this gap. It derives from "classical" equations for neural masses by reformulating these equations in a form which allows to show the existence of excitable chains for an open set of parameter values [Köksal Ersöz E, Aguilar C, Chossat P, Krupa M, Lavigne F. 2020. "Neuronal mechanisms for sequential activation of memory items: Dynamics and reliability." *PLOS ONE* 15(4): e0231165].

Generalization to more general graphs, with multiple branches and/or closed loops, proceeds along the same lines [Köksal Ersöz E, Chossat P, Krupa M, Lavigne F. 2022. "Dynamic branching in a neural network model for probabilistic prediction of sequences." *J Comput Neurosci* 50, 537–557].

I will also show how the model can give a cue to how decision strategy may proceed in a suddenly changing environment where synaptic adaptation is too slow to operate.

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24 Jul
10:45am

Quiver symmetry in network dynamical systems

Bob Rink

Vrije Universiteit Amsterdam

Dynamical systems often admit geometric properties that must be taken into account when studying their behavior. We show that many such properties can be encoded by means of quiver representations. These properties include classical symmetry, hidden symmetry, and feedforward structure, as well as subnetwork and quotient relations in network dynamical systems. A quiver equivariant dynamical system consists of a collection of dynamical systems with maps between them that send solutions to solutions. We prove that such quiver structures are preserved under Lyapunov–Schmidt reduction, center manifold reduction, and normal form reduction. This is joint work with Sören von der Gracht and Eddie Nijholt.

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24 Jul
11:30am

Nontransverse Poincaré sections and matched singularities

David Chillingworth

University of Southampton

Investigating dynamics of a nonsingular flow through the intersection of its orbits with a Poincaré section becomes complicated when the section is not transverse to the flow. Geometry of singularities then plays a key role in reinterpreting the dynamics as a hybrid system. An unexpected pairing of singularity types emerges that provides a general framework for reconstructing dynamics, applicable also to boundaries and obstacles.

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24 Jul
1:45pm

Rotating convection and flows with horizontal kinetic energy backscatter

Jens Rademacher

Universität Hamburg

Numerical simulations of large scale geophysical flows often require unphysically strong dissipation for numerical stability. A popular scheme to restore energetic balance is horizontal kinetic energy backscatter. We consider a continuum formulation with momentum equations augmented by a backscatter operator in e.g. rotating Boussinesq equations. Consistent with numerical observations, it turns out that the injected energy can accumulate in certain scales. We discuss the occurrence of this phenomenon in specific plane waves and related bifurcations in the presence of bottom drag.

Finite switching dynamics near heteroclinic networks

24 Jul
2:30pm
~~4:00pm~~

Sofia Castro
Universidade do Porto

This talk discusses the dynamics near a heteroclinic network, that is, near a connected union of two or more heteroclinic cycles. Near a node/equilibrium which belongs to more than one cycle, trajectories can follow one of the outgoing connections at this node and therefore, follow different cycles.

Switching dynamics refers to the fact that trajectories can 'switch' from following one of the cycles to following a different one at such a node. Switching dynamics can be simple or very complex depending on the possible choices that are available. I shall show that, near a heteroclinic network such that the Jacobian matrix at each node has only real eigenvalues, only finite switching exists. This means that not all arbitrary combinations of paths along the network can be followed by nearby trajectories.

In spite of this result near such networks interesting dynamics can be observed, as some game theory problems illustrate.

CANCELLED Dynamics and Symmetry applied to theoretical problems in neural nets and statistical learning

~~24 Jul~~
~~2:30pm~~

Michael Field
University of California, Santa Barbara

We describe results in dynamics, symmetry and geometric analysis that have applications to theoretical computer science and statistical learning. The emphasis throughout will be on the mathematics and topics related to the theme of the conference. However, there will be the odd aside about the problems from statistical learning and neural networks that are the motivation for this work.

Brief background. A characteristic of machine learning is that the methods generally work rather well in spite of theoretical predictions to the contrary. In particular, the optimization involved in the learning process is highly non-convex and so, from a general perspective, should not work well. Currently, much of the existing theory applies to limiting regimes. For example, in 2-layer networks, the thermodynamic limit from statistical physics , mean field theory, spin glasses, optimal control, Neural Tangent Kernel, compositional kernels, etc. These models typically represent limiting regimes where either the number of inputs or the number of neurons goes to infinity.

We address the highly non-convex optimization landscape in the natural (finite) regime and indicate how it is possible to give precise quantitative information about families of spurious minima (local not global minima) and their Hessian spectra, distinguish between different types of spurious minima, and understand the geometry involved in the creation and annihilation of spurious minima. Some of these questions have connections with old and familiar problems (such as quantifying how and why symmetric problems often have symmetric solutions). Additionally, there is the surprising role played by the symmetric group (specifically, the *standard* representation of the symmetric group).

At a later date, I hope to provide a short set of accompanying notes for the talk that will include some references as well as background discussion of statistical learning relevant to the talk. These notes will be available on request after the meeting.

The work presented in the talk is the result of a collaboration with Yossi Arjevani (School of Engineering and Computer Science, Hebrew University, Israel).

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25 Jul
9:00am

PDE heteroclinicity and blow-up: a tale of two sisters in complex times

Bernold Fiedler and Hannes Stuke
Freie Universität Berlin

The global dynamics of the real dissipative quadratic heat equation

$$w_t = w_{xx} + 6w^2 - \lambda\,, \qquad (*)$$

and her younger time-reversible, complex-valued "Schrödinger" sister

$$\mathrm{i}\psi_s = \psi_{xx} + 6\psi^2 - \lambda\,. \qquad (**)$$

seem to have little in common.

Consider $0 < x < \frac{1}{2}$, with Neumann boundary conditions. By its gradient-like structure, all *real eternal* non-equilibrium orbits $w(r) = \Gamma(r)$ of (*) are heteroclinic among equilibria $w = W_n(x)$. Nonhomogeneous real W_n of Morse index $i(W_n) = n = 1, 2, 3, \ldots$ are rescaled real-valued Weierstrass elliptic functions $W_n(x) = -n^2\wp(nx + \frac{1}{2}\tau)$.

We show that *real heteroclinic orbits* $\Gamma(r + \mathrm{i}s)$ *are accompanied by finite-time blow-up*, via analytic extension to imaginary time s. In particular, there exist r_0 such that the sister $\psi(s) := \Gamma(r_0 + \mathrm{i}s)$ in (**), of $w(r) = \Gamma(r)$ in (*), blows up at some finite real time $\pm s^*$.

Technically, however, we have to assume asymptotically stable target equilibria W. We also have to except a discrete set of $\lambda > 0$, and are currently limited to unstable dimensions $n \leq 22$, or to fast unstable manifolds at W_n of dimensions $d < 1 + \frac{1}{\sqrt{2}}n$.

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25 Jul
10:30am
~~4:30pm~~

Folded node revisited

Martin Krupa
Université Côte d'Azur

The first part of the talk is an introduction to the use of blow up in singular perturbation theory and a review of the known dynamics near a generic folded node. The second part focuses on the existence of the weak canard. We point out a flaw in the 2001 proof of Szmolyan and Wechselberger. We obtain a complete proof, under a stronger assumption, by combining the ideas of Szmolyan and Wechselberger with an approach developed by Benoit.

CANCELLED New Hidden Symmetries in Equivariant Dynamical Systems

~~25 Jul 10:30am~~

Michael Dellnitz
Universität Paderborn

We investigate matrices $A \in \mathbb{R}^{n \times n}$ with respect to their equivariance properties: It is well known that the equivariance of A with respect to certain groups $\Sigma \subset \mathbf{O}(n)$ generically leads to the existence of multiple eigenvalues. We show that in this case A is (additionally) equivariant with respect to the action of a group $\Gamma(A) \simeq \prod_{i=1}^{k} \mathbf{O}(m_i)$ where $m_1, \ldots, m_k$ are the multiplicities of the eigenvalues $\lambda_1, \ldots, \lambda_k$ of A – even if Σ is finite. Moreover, $\Gamma(A)$ consists of all the matrices which commute with A, so that in particular $\Sigma \subset \Gamma(A)$. We discuss implications of this result for equivariant nonlinear dynamical systems. This way we are able to explain the existence of solutions of certain types which is induced by the action of "hidden symmetries" in $\Gamma(A) \setminus \Sigma$. This is joint work with Raphael Gerlach (Paderborn University) and Sören von der Gracht (Paderborn University).

Lifting of heteroclinic cycles in coupled cell systems

25 Jul 11:15am

Ana Dias
Universidade do Porto

Given a network and one of its synchrony subspaces, any coupled cell system with form consistent with the network, restricted to the synchrony subspace, defines a coupled cell system which is consistent with a smaller network – the quotient network of the original network by the synchrony subspace. Conversely, there is also a systematic method of enumerating all networks (lifts) with a given quotient network. In this talk we use this method to show examples of robust heteroclinic cycles and networks, occurring in coupled cell systems, that are lifts of simple robust heteroclinic cycles, arising at lower dimensional symmetric coupled cell systems. Joint work with M. Aguiar (Porto) and H. Ruan (Hamburg).

Bifurcation without hysteresis — two examples

25 Jul 1:30pm

Arnd Scheel
University of Minnesota

I'll present two "bifurcation" scenarios that share the common feature of an (almost) vertical bifurcation branch. In the first example, we look at the transition from a crystalline state to clustering in interacting particle systems. In the second example, we consider instability in spatially extended systems in large but finite domains in the presence of (advective) transport. The vertical bifurcation branch allows for "phase transitions" without hysteresis, which may be exploited in biological switches.

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25 Jul
2:15pm

Infinitesimal Homeostasis in Input-Output Networks

Martin Golubitsky
The Ohio State University

Homeostasis occurs when a system output remains approximately constant on variation of a system input. A standard example of homeostasis occurs when the internal body temperature of a warm-blooded mammal remains approximately constant on variation of the ambient temperature. We define infinitesimal homeostasis as points where the derivative of output with respect to input vanishes. Fred Nijhout, Mike Reed, and Janet Best observed that homeostasis occurs frequently in biochemical networks, and we noted that such homeostasis can often be found by searching for infinitesimal homeostasis.

In this talk we review how network architecture can lead to different kinds of infinitesimal homeostasis and discuss how a version of network pattern formation can be associated with each kind of infinitesimal homeostasis. This is joint work with Fernando Antoneli, Will Duncan, Joe Huang, Jaxian Jin, Ian Stewart, and Yangyang Wang.

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26 Jul
9:00am

Motifs and Hypermotifs: Classification and Bifurcations

Manuela Aguiar
Universidade do Porto

Motifs, small subnetworks that carry out specific functions and occur unusually often, are important building blocks of biological networks. Therefore, the classification of small excitatory-inhibitory networks and their dynamical analysis is a fundamental step in the understanding of the dynamics of biological networks and, consequently, in obtaining answers to important biological questions.

We classify connected 2-node and 3-node excitatory-inhibitory networks under various conditions. Using results on ODE-equivalence and minimality, we classify the ODE-classes and present a minimal representative for each ODE-class.

In the literature, the term *hypermotif* is used to designate the combination of elementary motifs. Two particular ways in which two motifs can be combined to give rise to a hypermotif are: (i) the two motifs share at least one node and (ii) the two motifs are linked through at least one edge.

We consider the combination of two motifs where they share one node, designated by a *coalescence*. Given a codimension-one steady-state bifurcation problem in a coalescence network $N = N_1 \circ N_2$ we ask whether the bifurcation condition for N is necessarily a bifurcation condition for at least one of the networks N_1 or N_2. If the answer is positive, the next question we ask is to what extent it is possible to predict the bifurcation branches for N having knowledge of the bifurcation branches for N_1 and/or N_2. We call this the *bifurcation coalescence problem*.

The talk includes joint work with Ana Dias (University of Porto), Ian Stewart (University of Warwick) and Pedro Soares (University of Lisbon).

Levy area and time-reversal symmetry

26 Jul
10:30am

Ian Melbourne

The University of Warwick

Levy area arises naturally in the interpretation of stochastic integrals in limits of deterministic dynamical systems (ODEs). In particular, Levy area governs the departure from the Stratonovich interpretation.

In certain situations, time-reversal symmetry implies zero Levy area and hence Stratonovich integrals. The literature on the ubiquity of this has been somewhat unclear and is often over-stated. Here we clarify precisely the restrictions on Levy area due to time-reversibility.

Joint work with Georg Gottwald.

Cluster Synchronization as a Higher-Order Effect in Hypernetworks

26 Jul
11:15am

Eddie Nijholt

Imperial College London

We present a new framework for describing ODEs that model coupled cell systems with higher-order interactions. This generalises the definition of admissibility for classical, dyadic networks as introduced and explored by Golubitsky, Stewart, Field and others. As such, we find corresponding notions of balanced partitions for synchronisation patterns, and generalise these to hypergraph fibrations. Contrary to the network setting though, we find that linear systems do not determine all robust synchronisation patterns in the case of hypernetworks. One instead has to consider nonlinear terms as well, and we give a formula for the degree of the polynomial admissible maps that determine robust synchronisation patterns, in terms of the order of the hypernetwork. We also give a class of examples for which this degree is optimal. This phenomenon gives rise to some intriguing generic bifurcation scenarios, where synchrony between two nodes is broken at an unusually slow rate in the bifurcation parameter. We end with some examples and rigorous results for these "reluctant synchrony breaking"bifurcations. This is joint work with Sören von der Gracht and Bob Rink.

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26 Jul
1:30pm

Lagrangian transport and mixing studies by trajectories and complex networks

Kathrin Padberg-Gehle
Leuphana Universität Lüneburg

Transport and mixing processes in fluid flows are crucially influenced by coherent structures and the characterisation of these Lagrangian objects is a topic of intense current research. Recently, computational approaches have been proposed to identify coherent sets directly from an ensemble of trajectories by means of a Lagrangian flow network, where the links are weighted according to spatio-temporal distances between tracer trajectories.

In this talk, we discuss some extensions to the network-based framework. First, we consider the long-term behavior of coherent flow features by an adaption of evolutionary spectral clustering methods. Second, we demonstrate how to study and quantify advective and diffusive mixing within the network approach. We apply the proposed methods to several example systems, including turbulent Rayleigh-Bénard convection.

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26 Jul
2:15pm

The quest for robust early warnings of bifurcations in noisy systems

Peter Ashwin
University of Exeter

There has been a lot of interest in applied nonlinear dynamics in finding "early warning signals" or "precursors" of bifurcations in cases where bifurcation parameters are drifting. This is particularly relevant for "critical transitions" where a bifurcation such as a fold leaves no nearby solutions as stable. In applications it is of great interest in potentially getting warning of tipping points ahead, for example in various parts of the earth's climate system. The idea of an early warning is compelling: by examining the behaviour of a nonlinear system with weak noise, find properties of the noise (critical slowing down) that indicates a tendency towards less stable solutions. The challenge is to understand when one can expect such a signal to be reliable and robust. I will describe ongoing collaborative work that aims to understand more precisely the circumstances when such early warning signals may fail to work.

Poster Session

Artificial Neural Networks for nonlinear System of Mosquito Epidemic Model through Levenberg Marqduardt Computing Technique

Tahir Nawaz Cheema
Pakistan Model Higher Secondary School for girls Saroke Cheema Wazirabad

t. b. a.

Langevin equation of fractional order with new chaotic dynamics

Zoubir Dahmani
University Blida 1

t. b. a.

DynamicalSystems.jl: featureful, accessible and welcoming, excellently-documented software for all of nonlinear dynamics

George Datseris
University of Exeter

DynamicalSystems.jl is an award-winning Julia software library for nonlinear dynamics and nonlinear timeseries analysis. Its main three goals are 1) to be a library for nonlinear dynamics in the literal sense, 2) to make the field of nonlinear dynamics accessible and reproducible, and 3) to fundamentally change the perception of the role of code in both scientific education as well as research.

The content of DynamicalSystems.jl covers the entire field of nonlinear dynamics, all in one place: delay embeddings, recurrence quantification, stability analysis, continuation, global stability and multistability, chaos, timeseries analysis, complexity measures, and the list goes on. We are not aware of any software, open or closed source, that comes anywhere near to the amount of content in DynamicalSystems.jl.

Perhaps most importantly, DynamicalSystems.jl is an open source community driven software. Anyone can be a developer, everyone is welcomed to contribute and put in there their new methods, making them accessible to the community instantly as runnable code when their paper is published.

Extra care has been taken so that DynamicalSystems.jl is accessible: the source code is simple and concise and the programming interfaces of the software have been designed to be easily extendable. This lowers the threshold for a new contributor. Additionally, because the software is written in Julia, turning on development for it is a matter of only 1 minute (literally, no exaggerations!).

Join us in this poster to get an introduction to DynamicalSystems.jl and its content, see example code run live during the poster presentation!

Basin Entropy in the Revtokamap

Pedro Haerter
Universidade Federal do Paraná

t. b. a.

How nonlinear dynamics induce spectral-like measures

Jochen Merker
HTWK Leipzig

On the poster, in generalization of the spectral theorem for selfadjoint linear operators it is shown, how nonlinear operators with an absolutely homogeneous convex potential (like e.g. the p-Laplacian Δ_p with potential $u \mapsto \frac{1}{p}||\nabla u||^p_{L^p}$) can be decomposed using spectral-like measures obtained by solving doubly nonlinear gradient-like evolution equations. The presented methods extend the case of one-homogeneous functional discussed by [Burger M, Gilboa G, Moeller M, Eckardt L, Cremers D. 2016. “Spectral Decompositions Using One-Homogeneous Functionals.” *SIAM Journal on Imaging Sciences* 9(3): 1374–1408; Bungert L, Burger M, Chambolle A, Novaga M. 2021. “Nonlinear spectral decompositions by gradient flows of one-homogeneous functionals.” *Analysis & PDE* 14(3): 823–860; Burger M. 2022. “Nonlinear eigenvalue problems for seminorms and applications.” *ICM 2022 July 6-14*, Vol VII, EMS: 5234–5255] and allow various further generalizations.

Matas A, Merker J. 2012. “Existence of weak solutions to doubly degenerate diffusion equations.” *Appl. Math.* 57: 43–69.

Merker J. 2024. “Towards a nonlinear generalization of spectral decomposition.” *Book of extended abstracts, International Conference on Mathematical Analysis and Applications in Science and Engineering ICMASC’24, ISEP Porto-Portugal, June 20 - 22, 2024.*

Analytic proof of chaos in dynamical models

Ivan Ovsyannikov
Constructor University Bremen

t. b. a.

Extreme Events Scaling in BTW Sandpile Model

Abdul Quadir
Aligarh Muslim University Aligarh

t. b. a.

Heat transport in rotating Rayleigh-Benard convection

Roland Welter
Universität Hamburg

General circulation models aim to accurately represent the motions of the atmosphere and ocean using equations from fluid dynamics. However, vertical accelerations are often set equal to zero in primitive equation models. This assumption yields a significant mathematical benefit (well-posedness, numerical stability, etc) and is partially justified from a physical viewpoint, since the vertical accelerations should be small compared to more dominant forces (gravity, etc). Of course, such an assumption cannot be entirely physically accurate, and often additional terms are included to account for the discrepancy. This procedure is known as parameterization, and there is not a consensus about a correct or optimal way to parameterize convection.

This poster will present recent results which develop a mathematically rigorous framework for studying vertical heat transport in turbulent convection. Starting from the paradigmatic Boussinesq-Oberbeck equations, heat transport is investigated via the HKC hierarchy of Galerkin truncated ODE models. The dynamics of these models are studied, and particular attention is given to stable values of heat transport, as well as the convergence across models where the models accurately represent the PDE. Implications for energetically consistent parameterization of convection will then be discussed.

Higher-order phase reduction of delay-coupled oscillators

Babette de Wolff
Vrije Universiteit Amsterdam

In many real-life networks systems, it takes a significant time for signals to travel from node to node, leading to time delays in the coupling. Experiments show that coupling delays have a crucial and often counterintuitive effect on collective phenomena, including the synchronisation behaviour of coupled oscillators.

The poster introduces a phase reduction technique for delay-coupled oscillators, which gives a systematic way to derive equations for the phases of coupled oscillators. The resulting phase model is lower dimensional than the original model (in fact, it is finite dimensional while the original model is infinite dimensional), which facilitates further analysis of synchronisation phenomena. Applying the technique to an illustrative example shows how including these higher-order terms yields more accurate predictions of synchronisation behaviour.

This is based upon joint work with Christian Bick and Bob Rink.

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