A variant of the inverted Lanczos method

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In this lecture we consider the problem to determine the smallest eigenvalue (or the lower part of the spectrum) of a symmetric and positive definite matrix $A \in \mathbb{R}^{(n,n)}$ by a variant of the Lanczos method.
Motivated by a paper of Melman on bounds of the extreme eigenvalues of real symmetric Toeplitz matrices and by the fact that these bounds can be interpreted as the extreme eigenvalues of the projection of the eigenvalue problem to a Krylov space of A^{-1} we study a variant of the Lanczos method which produces Ritz values of A from $\mathcal{K}_k(A^{-1}, u)$.
The method can be executed in a similar way as the inverted Lanczos method. An orthonormal basis $q_1, \dots, q_k$ of $\mathcal{K}_k(A^{-1}, u)$ with respect to the scalar product $\langle x, y \rangle_A := x^T A y$ can be determined by a three term recurrence relation, and with $Q := (q_1, \dots, q_k) \in \mathbb{R}^{(n,k)}$ the projected eigenproblem $Q^T A Q y = \lambda Q^T Q y$ is tridiagonal. It is slightly faster than the original Lanczos process at least as long as reorthogonalization is not required.

References

A. MELMAN, Bounds on the extreme eigenvalues of real symmetric Toeplitz matrices. SIAM J. Matr. Anal. Appl. 21, 362 - 378 (2000)
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