On the stability of a linear system with variable parameters

A. A. IGNATYEV & A. O. IGNATYEV
Institute for Applied Mathematics & Mechanics
R.Luxemburg Street, 74
Donetsk-830114, Ukraine
mila@budinf.donetsk.ua

Consider a linear system with variable parameters on the plane

$$\dot{x} = y, \quad \dot{y} = -g(t)x - f(t)y, \tag{1}$$

where the damping and rigidity coefficients $f(t)$ and $g(t)$ are continuous and bounded functions of the time t. Most of the theories examining a stability problem of the zero solution are based on the Lyapunov stability and instability theorems and the corresponding Lyapunov function is assumed as an energy-type function

$$V = \frac{1}{2}c_1(t)y^2 + \frac{1}{2}c_2(t)x^2,$$

where $c_1(t), c_2(t)$ are time variable functions. In [1], A. P. Merkin considered the case $c_1(t) = c_2(t) = 1$ and stability conditions were obtained only for constant f and g. An extension was done in [2] for periodic functions $f(t)$ and $g(t)$. By means of a Lyapunov function which is a quadratic form with respect to x and y, V.M.Starzhinsky [3] (assuming that $0 < l \le f(t) \le L,\ 0 < m \le g(t) \le M$) obtained sufficient conditions of asymptotical stability for the solution

$$x = 0, \quad y = 0 \tag{2}$$

of equation (1). They are written as restrictions to the constants l, L, m, M. In this paper sufficient asymptotic stability conditions of the solution (2) are obtained which are close to necessary and sufficient conditions of stability. We suppose that $g(t)$ is continuously differentiable and that the inequalities

$$|f(t)| < M_1, \quad |g(t)| < M_2, \quad |\dot{g}(t)| < M_3$$

hold for $t \in R_+ = [0; \infty)$.
Theorem. If the conditions

$$g(t) > \alpha_1 > 0, \qquad p(t) = \frac{1}{2}\frac{\dot{g}(t)}{g(t)} + f(t) > \alpha_2 > 0$$

are fulfilled, then the solution (2) of the differential equations (1) is uniformly asymptotically stable.

References

[1] A. P. MERKIN, Stability of Motion. *"Nauka", Moscow*, (1976).
[2] V.A.YAKUBOVICH AND V.M.STARZHINSKY, Linear differential equations with periodic coefficients and their applications. *"Nauka", Moscow*, (1972).
[3] V.M.STARZHINSKY, Sufficient conditions of stability of the mechanical system with one degree of freedom.*J.of Appl. Math. and Mech.* **16**(1952), 369-374.