

$|\cdot|_1$ -radial, positive definite functions

WOLFGANG ZU CASTELL

Mathematical Institute

University of Erlangen-Nuremberg

Bismarckstr. 1 1/2, 91054 Erlangen

e-mail: castell@mi.uni-erlangen.de

In 1938 Schoenberg gave his characterization of positive definite, ℓ_2 -radial functions in terms of the Fourier-Bessel transform. In 1983 Cambanis, Keener and Simons showed an analogue representation for positive definite, ℓ_1 -radial functions; i.e., functions which are radial w.r.t. the ℓ_1 -norm ball. Berens & Xu independently proved the same characterization and further showed how to regain the inverse Fourier transform of these functions using an additional transform involving a B-spline kernel.

We identify the ℓ_1 -analogue of the Fourier-Bessel transform as an integral transform with a hypergeometric ${}_1F_2$ -function as kernel. The kernel of the inverse transform can also be expressed in terms of hypergeometric functions. We further show that the results can be extended to a more general class of kernels by using Dirichlet splines instead of B-Splines.

References

H. BERENS & W. ZU CASTELL, Hypergeometric functions as a tool for summability of the Fourier integral, *Result. Math.* **34** (1998), 69-84.