The linear rational collocation method with iteratively optimized poles for two-point boundary value problems

Jean-Paul Berrut & Hans D. Mittelmann
Université de Fribourg et Arizona State University
Institut de Mathématiques, CH-1700 Fribourg/Pérolles, Suisse
Deparment of Mathematics, Tempe, Arizona 85287-1804, USA
e-mail: jean-paul.berrut@unifr.ch mittelmann@math.la.asu.edu

In the present work we propose, and discuss with numerical examples, an algorithm that improves upon the polynomial pseudospectral method for solving two-point boundary value problems (BVPs).
The version of the pseudospectral method we have in mind is that consisting of replacing the solution by an interpolating polynomial in Lagrangian form between well-chosen points and collocating at those same points. Since such points are vertical projections on the interval of points (nodes) equidistant or nearly equidistant on the circle, they accumulate in the vicinity of the extremities of the interval. As a consequence, the method is well-suited for solving problems whose solutions have boundary layers. Correspondingly, however, the center nodes are about equidistant, and farther from each other than the same number of equidistant nodes on the whole interval. As a consequence, the prseudospectral method has a hard time approximating solutions with large gradients (shocks) away from the extremities of their domain of definition.
We present here a method that improves on the pseudospectral (polynomial) method without any change in the interpolation/collocation points and preserves the infinite differentiability of the solution. The idea is to modify the trial functions by attaching a denominator to the polynomial so as to make it a rational interpolant. The denominator is then successively optimized in an iterative procedure, with each step consisting of the solution of two problems: an optimization of the denominator for given values of the approximation to the solution at the interpolation points, and a collocation in the linear space of the rational interpolants with the just obtained, fixed denominator, to obtain new appoximate values of the solution.