# 1 Overview talks

**Dror Bar-Natan**
***A quick introduction to Khovanov homology***

I will tell the Kauffman bracket story of the Jones polynomial as Kauffman told it in 1987, then the Khovanov homology story as Khovanov told it in 1999, and finally the "local Khovanov homology" story as I understood it in 2003. At the end of our 90 minutes we will understand what is a "Jones homology", how to generalize it to tangles and to cobordisms between tangles, and why it is computable relatively efficiently. But we will say nothing about more modern stuff - the Rasmussen invariant, Alexander and HOMFLYPT knot homologies, and the categorification of sl2 and other Lie algebras.

**Sergei Gukov**
***Introduction to the volume conjecture***

**Peter Teichner**
***Survey of functorial field theories***

We discuss a mathematical approach to field theories initiated by Atiyah (for TFTs) and Segal (for CFTs). The emphasis will be on an extension of the Atiyah-Segal language by higher categories, modeling the physical locality of fields.

# 2 Research talks

**Jørgen Andersen**
***The geometric construction of the Reshetikhin-Turaev Topological Quantum Field Theory***

In this talk, we will discuss the geometric construction of the Reshetikhin-Turaev Topological Quantum Field Theory using the geometric quantization of the moduli spaces of flat connections on two dimensional surfaces. We will then discuss various results on the large level asymptotics of these theories.

**Dror Bar-Natan**
***Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant***

Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space - hoops can be composed like in $\pi_1$, balloons like in $\pi_2$, and hoops "act" on balloons as $\pi_1$ acts on $\pi_2$. We will observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.

We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word) -valued invariant Z of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that Z is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of Z is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, that's a wonderful playground.

**John Barrett**
***Gray categories and their diagrams***

In this talk I will report on the results of a research project with Catherine Meusburger and Gregor Schaumann. The results concern coherence for Gray categories and properties of Gray category diagrams. If time permits, I will briefly sketch the possible application of this theory to defects in TQFT.

**Nils Carqueville**
***Defects and adjunctions in Landau-Ginzburg models***

Two-dimensional topological field theories with defects are in general expected to be described by certain bicategories with adjoints. This can be worked out very explicitly and elegantly in the case of Landau-Ginzburg models. I shall explain the construction (which is joint work with Daniel Murfet) and indicate some of its applications, including a unified perspective on open/closed TFTs, a simple proof of the general Cardy condition, and (in joint work with Ingo Runkel) a generalised orbifold construction.

**Alberto Cattaneo**
***Classical and quantum Lagrangian field theories on manifolds with boundaries***

Classical and quantum field theories may be thought of as appropriate functors from (some version of) the cobordism category. At the quantum level this was proposed by Segal as an axiomatization. Incarnations of this exist for nonperturbative topological field theory (by Witten following Atiyah's version of the axioms for TFTs) as well as in one and two dimensional field theories. This talk (based on joint work with Mnev and Reshetikhin) will give an introduction to the classical version and to the Batalin-Vilkovisky version, which forms the starting point for the perturbative quantization. The possibility of including boundaries of boundaries (and so on) naturally yields to a Lurie-type description. Eventually, one might be able to reconstruct perturbative quantum theories on manifolds by gluing simple pieces together. Work in progress on this will be presented.

**Tudor Dimofte**
***Spin Networks, Teichmüller Theory, and RG Domain Walls***

I will discuss an interesting class of 3-manifolds with boundary, endowed with an ideal triangulation. Chern-Simons-like partition functions on these manifolds, calculated using the ideal triangulation, provide a new way to understand spectra of geodesic length operators in Teichmüller theory. On the other hand, by compactifying M5 branes on these manifolds and associating 3d N=2 SCFT's to them, one can construct domain walls that implement RG flow for 4d SCFT's.

**Sergei Gukov**
***Knot Homologies from Quantization of Moduli Space***

The generalized volume conjecture states that "color dependence" of the colored Jones polynomial is governed by an algebraic variety, the zero locus of the A-polynomial (for knots) or, more generally, by character variety (for links or higher-rank quantum group invariants). This relation, based on SL(2,C) Chern-Simons theory, explains known facts and predicts many new ones.

In particular, since the colored Jones polynomial can be categorified to a doubly-graded homology theory, one may wonder whether the generalized (or quantum) volume conjecture admits a natural categorification. In this talk, I will argue that the answer to this question is "yes" and introduce a two-parameter deformation of the A-polynomial that describes the "color behavior" of the HOMFLY homology, much like the ordinary A-polynomial does it for the colored Jones polynomial. This deformation, called the super-A-polynomial, is strong enough to distinguish mutants, and its most interesting properties include relation to knot contact homology and knot Floer homology.

**Theo Johnson-Freyd**
***Nonperturbative integrals, imaginary critical points, and homological perturbation theory***

The method of Feynman diagrams is a well-known example of *algebraization* of integration. Specifically, Feynman diagrams algebraize the asymptotics of integrals of the form $\int f \exp(s/\hbar)$ in the limit as $\hbar \to 0$ along the pure imaginary axis, supposing that $s$ has only nondegenerate critical points. (In quantum field theory, $s$ is the "action," and $f$ is an "observable.") In this talk, I will describe an analogous algebraization when $\hbar = 1$ — no formal power series will appear — and $s$ is allowed degenerate critical points. Nevertheless, some features from Feynman diagrams remain: I will explain how to algebraically "integrate out the higher modes" and reduce any such integral to the critical locus of $s$; the primary tool will be a *homological* form of perturbation theory (itself almost as old as Feynman's diagrams). One of the main new features in nonperturbative integration is that the critical locus of $s$ must be interpreted in the *scheme-theoretic* sense, and in particular imaginary critical points do contribute. Perhaps this will shed light on questions like the Volume Conjecture, in which an integral over SU(2) connections is dominated by a critical point in $\mathrm{SL}(2,\mathbb{R})$.

**Rinat Kashaev**

***On some calculations within Teichmüller TQFT***

A sufficient condition for Teichmüller TQFT to associate an absolutely convergent partition function to a one cusp 3-manifold is the existence of an ideal triangulation admitting an angle structure. Nonetheless, there are examples of ideal triangulations which do not admit any angle structure but their Teichmüller TQFT partition functions can be calculated as conditionally convergent integrals. I will consider two examples of such calculations corresponding to the trefoil complement and the figure-eight sister.

**Scott Morrison**

***Khovanov homology for 4-manifolds***

I'll explain a recipe (joint with Kevin Walker) for define vector-space valued invariants of 4-manifolds from Khovanov homology. There are some obstacles, forcing us to work in characteristic 2 for now. The blob complex (also joint with Kevin Walker) generalizes this to a sequence of homology groups for each 4-manifold (the original invariant being $H_0$), which may reflect the familiar long exact sequences computing the Khovanov homology of links.

**Hitoshi Murakami**

***The colored Jones polynomial, the Chern-Simons invariant, and the Reidemeister torsion of a knot***

I will talk about a relation of the colored Jones polynomial to the Chern-Simons invariant and the Reidemeister torsion of a representation of the fundamental group of the complement of a knot to SL(2;C).

**Satoshi Nawata**

***Super-A-polynomials of twist knots***

The superpolynomial $P(K; a, q, t)$ is defined as a Poincare polynomial of the triply-graded homology theory. The superpolynomial unifies many polynomial and homological invariants of knots which can be obtained from it via various special limit or via applying differentials. We propose the form of the colored superpolynomials for a class of twist knots $K_p$ where $p$ denotes the number of full-twists. By using this formula, we check the refined version of the volume conjecture and the AJ conjecture of twist knots. Furthermore, in the case of twist knots, we can varify the conjecture by Agangic and Vafa that the Q-deformed A-polynomials are identified with augmentation polynomials of knot contact homology.

**Hendryk Pfeiffer**

***Canonical Bases for objects of SL(2) Fusion Categories***

Let $r = 2, 3, \ldots$, $q$ be a primitive $2r$-th root of unity, and consider the usual finite-dimensional quotient of $U_q(sl_2)$. Its category of tilting modules modulo negligible morphisms is a fusion category. By $V_0, \ldots, V_{r-2}$, we denote representatives of the isomorphism classes of the simple objects.

We represent each $V_j$ by a finite-dimensional vector spaces over $\mathbb{Q}(q)$ that is equipped with a canonical basis. In these bases, 'half' of all inclusions and projections involved in the fusion $V_j \otimes V_1 \cong V_{j-1} \oplus V_{j+1}$, $1 \leq j \leq r-3$, have only integer coefficients. Whereas over $\mathbb{Q}(q)$, tensor products decompose into direct sums of simple objects, over $\mathbb{Z}$ we find composition series with the corresponding simple factors.

**Gregor Schaumann**
***Towards a tricategory of defects for the Turaev-Viro model***

Starting from a geometric analysis of a 3-d tft with oriented defects, I will specify the kind of tricategory with duals that is expected to underly a 3-d tft with defects based on the Turaev-Viro construction. The tricategory of bimodule categories with the additional structure of module traces constitutes a candidate for such a tricategory. I will describe module traces, discuss the existence and uniqueness problem and show how they apply in the construction of the necessary duality structures. Part of this is a joint project with Catherine Meusburger and John Barrett.

**Ed Segal**
***Chern characters for matrix factorizations via non-commutative translation***

If we study the B-twist of a Landau-Ginzburg model with superpotential W, then it's well-known that the category of B-branes is the category of matrix factorizations of W, and also that the closed state space is given by the Jacobi ring of W. It's therefore reasonable to conjecture that the Jacobi ring is equal to the Hochschild homology of the category of matrix factorizations, since this is the universal closed sector compatible with this open sector. This talk will be about my failed attempt to prove this conjecture (which is now a theorem of Dyckerhoff). As a positive outcome of my attempt we get an explicit formula for the n-point boundary-bulk propagators in the model, and as a corollary we can derive the famous Kapustin-Li formula. We produce this formula using an operation of 'non-commutative translation' on the category of B-branes.

**Peter Teichner**
***Invertible field theories and differential cohomology***

It is well known that differential cocycles give topological terms in a classical Sigma model. We will outline a proof that this is the only way to produce (higher categorical, invertible, smooth) field theories. This requires a family version of the extended Atiyah-Segal notion of TFTs.

**Miguel Tierz**
***Matrix models in Chern-Simons theory***

We give an introductory overview of matrix models in Chern-Simons theory and q-deformed 2d Yang-Mills theory and discuss the relationship between the two. We will show how the same matrix models are also useful in problems of enumerative geometry and we will also present recent generalizations of the matrix models, involving the Macdonald measure, that describe refined Chern-Simons theory.

**Masahito Yamazaki**
***Quantum Dilogarithms and Elliptic Gammas in Gauge Theories***

Classical and quantum dilogarithm functions are known to play crucial roles in the theory of hyperbolic 3-manifolds and non-compact Chern-Simons theory, and more recently in the physics of 3d supersymmetric gauge theories. What is probably less known is that a quantum dilogarithm function could be "lifted" to another function, the elliptic gamma function. I will describe the role of elliptic gamma functions in a particular class of 4d quiver gauge theories defined geometrically and discuss their reduction to quantum dilogs. Along the way I will comment on the relations with a number of math/physics ideas, including AGT-type correspondence, integrable spin systems (akin to spin networks), dimer models, discrete conformal transformations, totally positive Grassmannians and cluster algebras.