Mathematics Department
Prof. Dr. J. Behrens

WiSe 2021/2022

Mathematics III Exam
(Module: Differential Equations I)

February 28, 2022

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in block letters in the designated fields following. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the Central Examination Office of TUHH verifies my official admission before the exam's beginning in retrospect.

(Signature)

Exercise	**Points**	**Evaluater**
1		
2		
3		
4		
5		

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Exercise 1: (3+2 points)

a) Solve the initial value problem

$$y' - y = 2 \quad \text{with} \quad y\,(0) = 3.$$

b) Compute the general solution of Euler's differential equation

$$x^2 y'' + xy' - 4y = 0\,.$$

Hint: There exist solutions of the form $y(x) = x^\alpha$ with $\alpha \in \mathbb{R}$.

Exercise 2: (5 points)

Compute the general solution of the following system of differential equations

$$\boldsymbol{y}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \boldsymbol{y} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Hint: Note that $\lambda^2 - 4\lambda + 3 = (\lambda - 1)(\lambda - 3)$.

Exercise 3: (4 points)

Solve the initial value problem

$$y'' + 2y' - 3y = 7 - 6x\,, \quad y(0) = 0\,,\; y'(0) = 3\,.$$

Hint: Note that $\lambda^2 + 2\lambda - 3 = (\lambda - 1)(\lambda + 3)$.

Exercise 4: (3 points)

Compute the real-valued general solution of the linear differential equation

$$y'' + 9y = 0.$$

Use this solution to obtain all solutions for the corresponding boundary value problem with boundary values $y'(0) = 6$ and $y\left(\frac{\pi}{6}\right) = 2$.

Exercise 5: (3 points)

Compute all equilibria for the nonlinear system of differential equations of first order

$$\begin{aligned} \dot{x} &= y - 4 \\ \dot{y} &= 4y(x-1) \end{aligned}$$

and determine their stability properties.