

Differential Equations I for Students of Engineering Sciences

Sheet 6 (home)

Exercise 1:

Compute the general solution of the differential equation

$$y'' + 16y = 16 \quad \text{with} \quad 0 \leq x \leq \frac{\pi}{8}.$$

Then, rewrite the equation into a system of first order and employ the shooting matrix to obtain the solutions for the following boundary conditions:

- a) $4y(0) + y' \left(\frac{\pi}{8} \right) = 4$ and $y \left(\frac{\pi}{8} \right) = 0$,
- b) $y(0) = 1$ and $y' \left(\frac{\pi}{8} \right) = 1$,
- c) $y(0) + y'(0) = 0$ and $y \left(\frac{\pi}{8} \right) + y' \left(\frac{\pi}{8} \right) = 0$.

Exercise 2:

- a) Compute the equilibrium of the following system of differential equations, investigate its stability, determine its type, compute the general solution of the system of differential equations and sketch the phase plot:

$$\begin{aligned} \dot{x} &= y - x/2 - 1, \\ \dot{y} &= -x - y/2 + 3. \end{aligned}$$

- b) Determine all stationary solutions (equilibria) of the system of differential equations

$$\begin{aligned} y_1' &= y_1 - y_2 \\ y_2' &= (y_1 - 1)(y_2 - 3), \end{aligned}$$

and investigate their stability properties by (local) classification.