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On the Mathematics of Tunnel Fires

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Overview:

- Introduction
- Modelling
- Analysis
  - Stationary Problem
  - Transient Problem
  - Stability
- Validation of the Model
  - Tunnel on the A22
- Tunnel Networks

Background:

- 2006 - Viamala Tunnel: 9 dead
- 2005 - Frejus Tunnel: 2 dead
- 2001 - Baltimore: 0 dead
- 2001 - Gotthard Tunnel: 11 dead
- 2000 - Kaprun: 155 dead
- 1999 - Mont Blanc Tunnel: 39 dead
- 1999 - Tauern Tunnel: 12 dead
- 1996 - Euro Tunnel: 0 dead
- 1995 - Baku: 300+ dead

Tunnel fires:

Typical questions:

- In which direction is the smoke going?
- How fast?
- What are the temperatures in the tunnel?
- Which is the right direction to escape?
- How can mobile ventilation systems be used?

Available software tools: (Oleinick & Carpenter ('03))

- Zone models (indoor fires)
- CFD Tools: Smartfire, Veti, Fire Dynamics Simulator, Solvent, Hitecosp, ...
- One dimensional tools: NewVendis, Camatt, Sprint,...

Problem (for long tunnels):

Relations: height/length $= 10\text{m}/10\text{km} \ll 1$

- 3d (CFD) is (very) expensive.
- 3d needs many data (which often are not known).
- 3d needs a good "postprocessing".
- 3d needs a sofisticated turbulence model.
- in 3d layered air/smoke flows can be described.
- 1d modells (for mean values in the cross-section) are a good alternative.

Problem :

Chemistry of the fire:

- fire is modeled as a heat source.

2 more Problems :

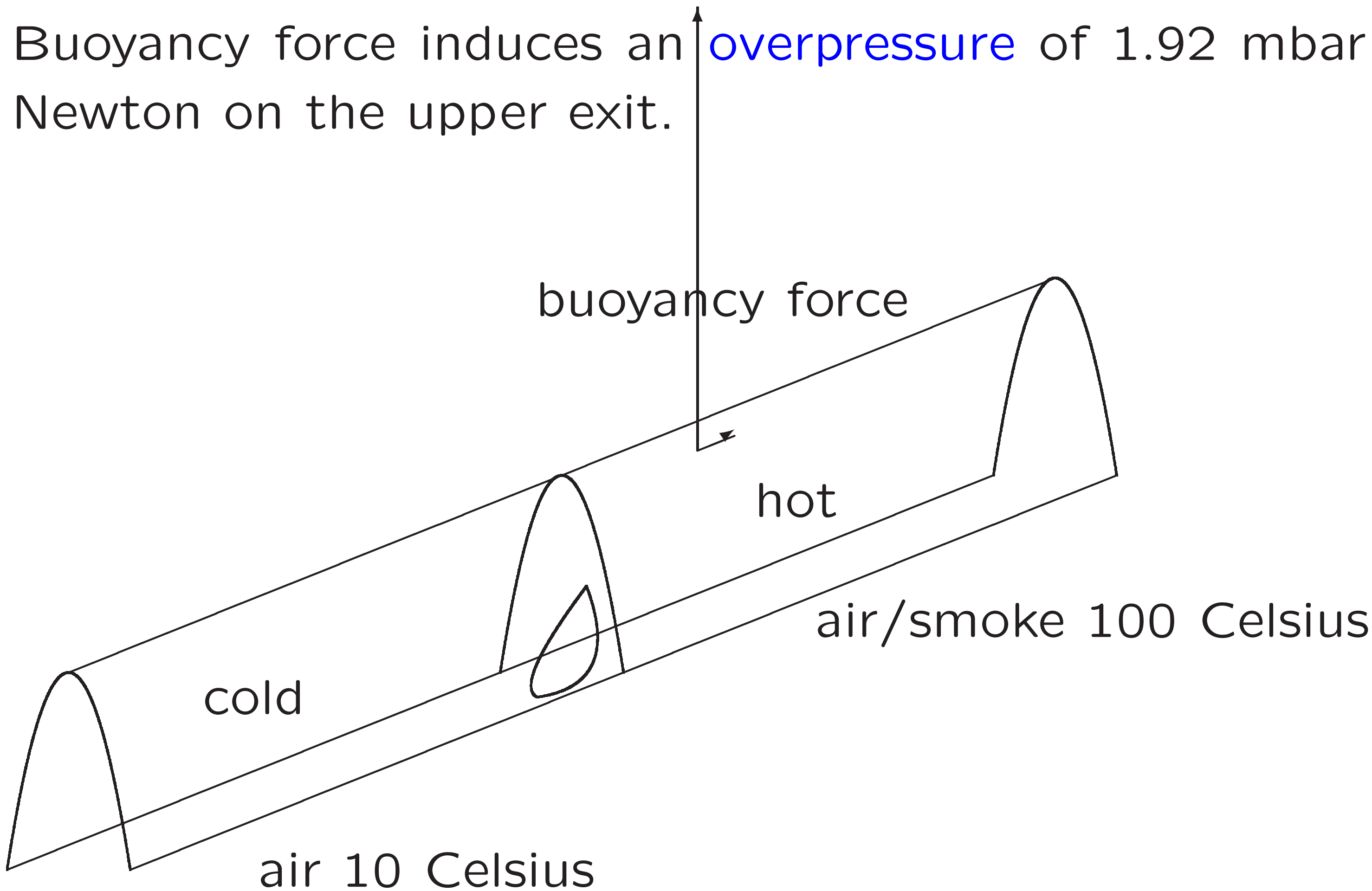
Velocities: $\approx 0 - 20$ m/s

Temperatures: $10^\circ - 2000^\circ$ Celsius

- Small Mach number $\Rightarrow$ incompressible model?
- Energy transport in an incompressible model $\Rightarrow$ Boussinesq approximation?
- Bussinesque approximation is only valid for small temperature differences
- Big temperature differences $\Rightarrow$ compressible model?
- Compressible models have many problems in the small mach number regime

Model tunnel: $4\,km$ long, $100\,m^2$ crossection, 3% slope, in the middle a fire.

Buoyancy force induces an overpressure of 1.92 mbar or 19200 Newton on the upper exit.



Starting point: $1d$ compressible Navier–Stokes equations

$$\begin{aligned}
&\tilde{\rho}_{\tilde{t}} + (\tilde{\rho}\tilde{u})_{\tilde{x}} = 0,\\
&\tilde{u}_{\tilde{t}} + \tilde{u}\tilde{u}_{\tilde{x}} + \frac{1}{\tilde{\rho}}\tilde{p}_{\tilde{x}} = \eta\frac{1}{\tilde{\rho}}\tilde{u}_{\tilde{x}\tilde{x}} + \boxed{\tilde{p}_l} + \tilde{f},\\
&(c_v\tilde{\rho}\tilde{T})_{\tilde{t}} + (c_v\tilde{u}\tilde{\rho}\tilde{T})_{\tilde{x}} + \tilde{p}\tilde{u}_{\tilde{x}} = \lambda\tilde{T}_{\tilde{x}\tilde{x}} + \tilde{q} - \tilde{\rho}\tilde{u}\tilde{f} - \eta\tilde{u}\tilde{u}_{\tilde{x}\tilde{x}}.
\end{aligned}$$

Variables: $\tilde{x}, \tilde{t}, \tilde{\rho} = \tilde{\rho}(\tilde{x}, \tilde{t}), \tilde{u} = \tilde{u}(\tilde{x}, \tilde{t}), \tilde{p} = \tilde{p}(\tilde{x}, \tilde{t}), \tilde{T} = \tilde{T}(\tilde{x}, \tilde{t})$

- Viscosity η, heat conductivity λ, specific heat c_v
- $\tilde{f}$ (gravitational + external force), heat source $\tilde{q}$ (fire)
- ideal gas law: $\tilde{p} = R\tilde{\rho}\tilde{T}$
- pressure loss in the tunnel $\tilde{p}_l$

Dimensional analysis: Set $\tilde{a} = a_r \cdot a$

Quantity	Unit	Referencevalue	Typical Referencevalue
t	s	$t_r = L/u_r$	900 s = 15 min
x, y, z	m	L	10^3-10^4 m
Tunnelheight	m	d	10 m
A (crosssection)	m^2	A_r	10^2 m^2
u	m s^{-1}	u_r	1 m s^{-1}
ρ	kg m^{-3}	ρ_r	1.2 kg m^{-3}
p	kg m^{-1} s^{-2}	p_r	1 bar = 10^5 kg m^{-1} s^{-2}
f	m s^{-2}	f_r	10 m s^{-2}
T	K	$T_r = \frac{p_r}{\rho_r R}$	300 K
q	W m^{-3}	q_r	10^5–10^6 W m^{-3}
R	m^2 s^{-2} K^{-1}		287 m^2 s^{-2} K^{-1}
c_p	m^2 s^{-2} K^{-1}		1005 m^2 s^{-2} K^{-1}
η	kg m^{-1} s^{-1}		18 x 10^{-6} kg m^{-1} s^{-1}
λ	kg m s^{-3} K^{-1}		25 x 10^{-3} kg m s^{-3} K^{-1}

Machnumber M

$$M^2 = \frac{\rho_r u_r^2}{\gamma p_r} = 8.6 \cdot 10^{-6}$$

Scaled compressible NS equations I:

$$
\begin{aligned}
\rho_t + (\rho u)_x &= 0,\\
u_t + u u_x + \left(\frac{1}{\gamma M^2}\right)\frac{1}{\rho} p_x &= \eta \frac{1}{\rho} u_{xx} + \boxed{\overline{p_l} p_l} + \overline{f} f,\\
(\rho T)_t + (u\rho T)_x + (\gamma - 1) p u_x &= \lambda T_{xx} + \boxed{\overline{q} q}\\
&\quad - M^2 \gamma(\gamma - 1)\overline{f}\, \rho u f - M^2 \eta u u_{xx}
\end{aligned}
$$

Variables: (longitudinal) space x, time t, density $\rho = \rho(x,t)$, velocity $u = u(x,t)$, temperature $T = T(x,t)$, pressure $p = p(x,t)$

- viscosity η, heat conductivity λ, adiabatic Constant γ
- Mach number M
- ideal gas law: $p = R\rho T$

Scaled compressible NS equations II:

$$
\begin{aligned}
\rho_t + (\rho u)_x &= 0, \\
u_t + u u_x + \left(\frac{1}{\gamma M^2}\right)\frac{1}{\rho} p_x &= \eta \frac{1}{\rho} u_{xx} + \boxed{\overline{p_l} p_l} + \overline{f} f, \\
(\rho T)_t + (u \rho T)_x + (\gamma - 1) p u_x &= \lambda T_{xx} + \boxed{\overline{q} q} \\
&\quad - M^2 \gamma (\gamma - 1) \overline{f}\, \rho u f - M^2 \eta u u_{xx}
\end{aligned}
$$

- gravitational force $\overline{f} \cdot f = g \cdot (-\sin \alpha)$ (slope profile $\alpha(x)$)
- heat source $q = q(x, t)$ (fire)
- pressure loss in the tunnel $\overline{p_l} p_l = -\frac{\hat{\xi}}{d} \frac{u|u|}{2}$ (turbulence)

Scaled compressible NS equations III:

$$\begin{aligned}
\rho_t + (\rho u)_x &= 0, \\
u_t + u u_x + \left(\frac{1}{\gamma M^2}\right)\frac{1}{\rho} p_x &= \eta \frac{1}{\rho} u_{xx} + \overline{p_l} p_l + \overline{f} f, \\
(\rho T)_t + (u \rho T)_x + (\gamma - 1) p u_x &= \lambda T_{xx} + \overline{q} q \\
&\quad - M^2 \gamma (\gamma - 1) \overline{f} \rho u f - M^2 \eta u u_{xx}
\end{aligned}$$

Limit $\eta \to 0$ $\lambda \to 0$

Asymptotics from Navier-Stokes to Euler

(Gilbarg '51) (for travelling waves)

(I.G., P.Szmolyan '93) (for travelling waves with combustion)

(Wagner '89) (with combustion)

Scaled compressible NS equations IV:

$$
\begin{aligned}
\rho_t + (\rho u)_x &= 0, \\
u_t + u u_x + \left(\frac{1}{\gamma M^2}\right)\frac{1}{\rho} p_x &= \eta \frac{1}{\rho} u_{xx} + \overline{p_l} p_l + \overline{f} f, \\
(\rho T)_t + (u \rho T)_x + (\gamma - 1) p u_x &= \lambda T_{xx} + \overline{q_w} q_w + \overline{q} q \\
&\quad - M^2 \gamma (\gamma - 1) \overline{f}\, \rho u f - M^2 \eta u u_{xx}
\end{aligned}
$$

Charakteristic values:

Mach number $M \approx 10^{-3} \;\Rightarrow\; \varepsilon = \gamma M^2 \ll 1$

Small Mach number asymptotics:

$$p = p_0 + \varepsilon p_1 + O(\varepsilon^2)$$

momnetum balance gives

$$(p_0)_x = 0 \quad \Rightarrow \quad p_0 = p_0(t)$$

Assumption: $p_0 = \text{const.}$ (leading order pressure) $T = \frac{p_0}{\rho}$

Initial boundary value problem (I.G., J.Struckmeier '02)

$$\begin{aligned} \rho_t + u\rho_x &= -\rho q, \\ u_t + uu_x + \frac{1}{\rho}(p_1)_x &= -\xi\frac{u|u|}{2} - f_d \sin\alpha \\ u_x &= q \end{aligned}$$

Initial values:

$$u(x,0) = u_0(x) \quad \rho(x,0) = \rho_0(x)$$

Boundary values:

$$\begin{aligned} p_1(0,t) &= p_{10}, \quad p_1(1,t) = p_{11} \\ u_x(0,t) &= u_x(1,t) = 0 \\ \rho(0,t) &= \rho_0 \; (u(0,t) > 0), \quad \rho(1,t) = \rho_1 \; (u(1,t) < 0) \end{aligned}$$

ρ density, u velocity, p_1 pressure (corrections)

$\alpha = \alpha(x)$ slope profile, $\xi = \xi(x)$ pressure loss, f_d scaled gravitational constant, $q = q(x,t)$ scaled heat source

Contains only one paramter (ξ) !

Asymptotical model in the 3d case: For $\overline{\eta}, \overline{\lambda}, \overline{f}, \overline{q} = O(1)$

$$
\begin{aligned}
\rho_t + \mathrm{div}(\rho u) &= 0, \\
u_t + u \cdot \nabla u + \frac{1}{\rho} \nabla p_1 &= \overline{\eta} \frac{1}{\rho} \Delta u + \overline{f} f, \\
\mathrm{div} u &= \frac{\overline{\lambda}}{\overline{\gamma}} \Delta \frac{1}{\rho} + \frac{\overline{q}}{\gamma p_0} q.
\end{aligned}
$$

(Majda '84, Embid '87)

Analysis

- Stationary problem (I.G. '02):

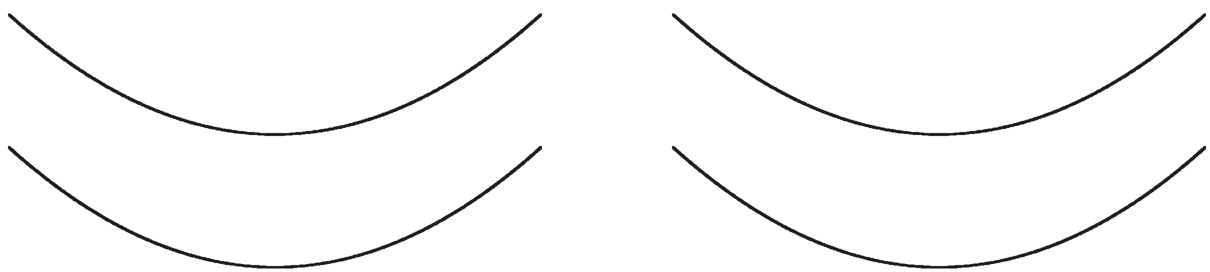
$$
\begin{aligned}
u\rho_x &= -\rho q, \\
u u_x + \frac{1}{\rho}(p_1)_x &= -\xi \frac{u|u|}{2} - f_d \sin\alpha \\
u_x &= q
\end{aligned}
$$

Boundary values:

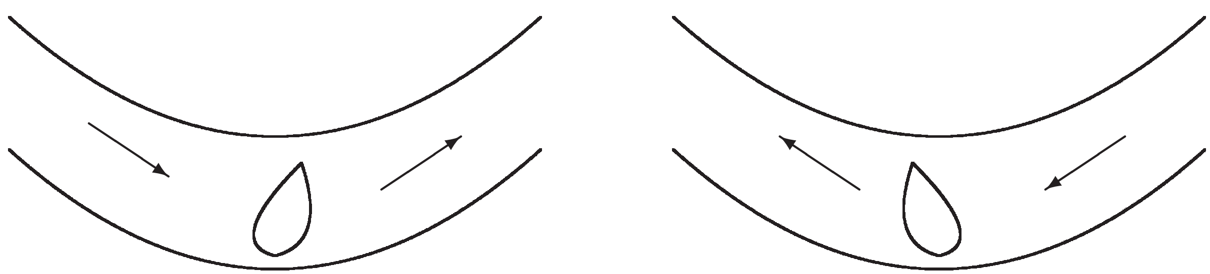
$$
\begin{aligned}
p_1(0) &= p_{10}, \quad p_1(1) = p_{11} \\
u_x(0) &= u_x(1) = 0 \\
\rho(0) &= \rho_0 \ (u(0) > 0), \quad \rho(1) = \rho_1 \ (u(1) < 0)
\end{aligned}
$$

There exist **multiple** (non-vacuum) solutions

No fire: air at rest



With fire: at least two possibilities



- Transient problem I:

  Reformulation

  with $u(x,t) = v(t) + \int_0^x q(y,t)dy = v(t) + Q(x,t)$

  and $I_f(t) = \int_0^1 f(x,t)\rho(x,t)dx$ we eliminate the pressure

  This gives a PDE for $\rho = \rho(x,t)$ and an ODE for $v = v(t)$,

$$\rho_t + (v+Q)\rho_x = -\rho q,$$

$$I\, v_t + I_q\, v + \int_0^1 \xi\rho \frac{(v+Q)|v+Q|}{2} dx = -I_{Q_t + Qq + f_d \sin\alpha} - p_l + p_r$$

  Initialdata:

$$v(0) = u_0(x) - \int_0^x q(y,0)dy \qquad \rho(x,0) = \rho_0(x)$$

  Boundary data:

$$\rho(0,t) = \rho_0 \; (u(0,t) > 0), \quad \rho(1,t) = \rho_1 \; (u(1,t) < 0)$$

Global existence- and uniqueness result

(I.G., H.Steinrück '06)

Solutions of the type:

$v \in C^1[0,T]$ but in ρ we have to admit discontinuities.

These are natural due to the inflow conditions.

Idea of the proof:

Fixed-point- argument in the ODE

Use estimates on the density from the PDE in the ODE

- Transient problem II:

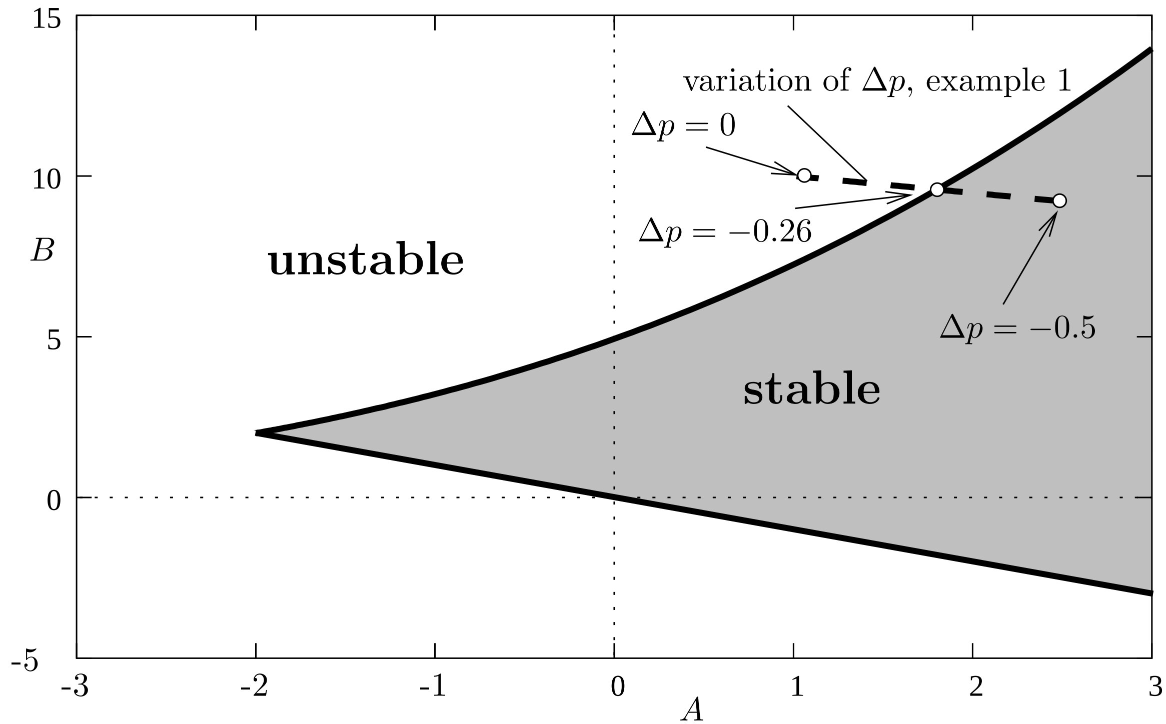
  Stability (I.G., H.Steinrück ’06)

  Stability of the solutions of the stationary problem as solutions of the transient problem,

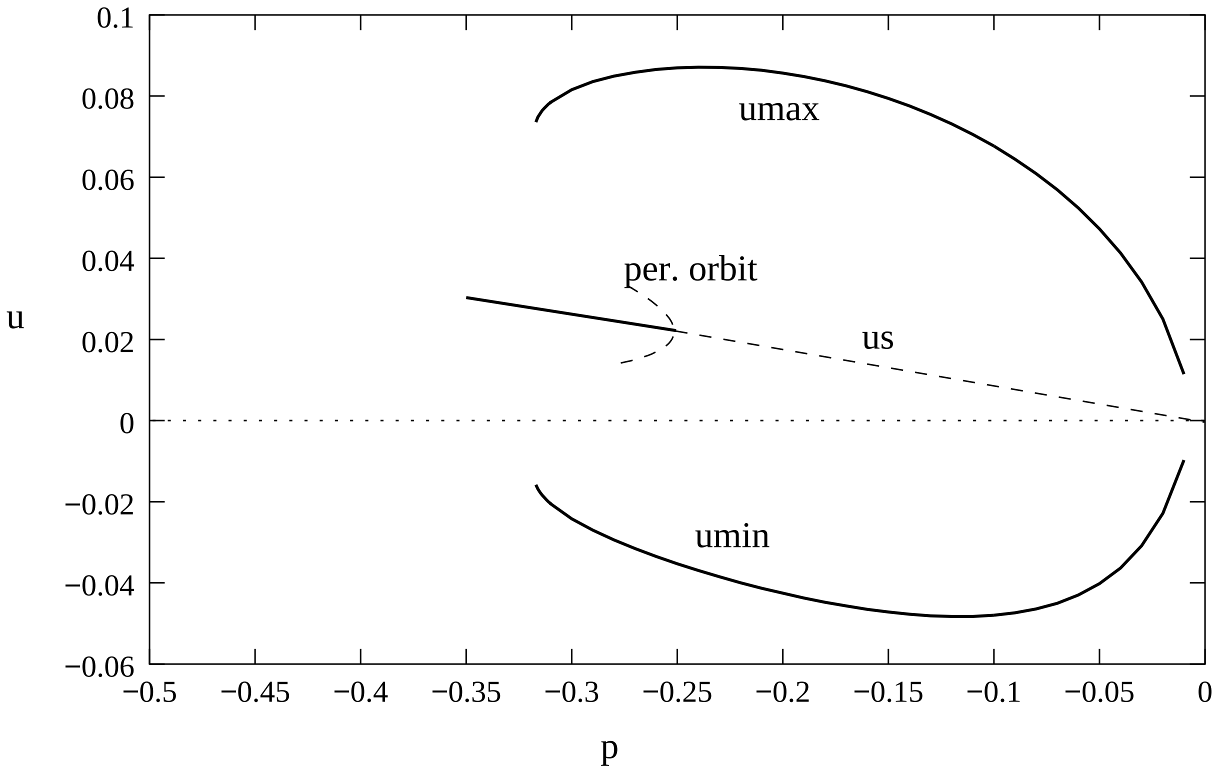
  Linear stabiltiy anaylsis gives a stability problem of a Volterra-Integro-Differential equation.

  Numerical bifurcation analysis (for example depending on the pressure difference at the boundaries).

Areas of stabilty



bifurcation diagram

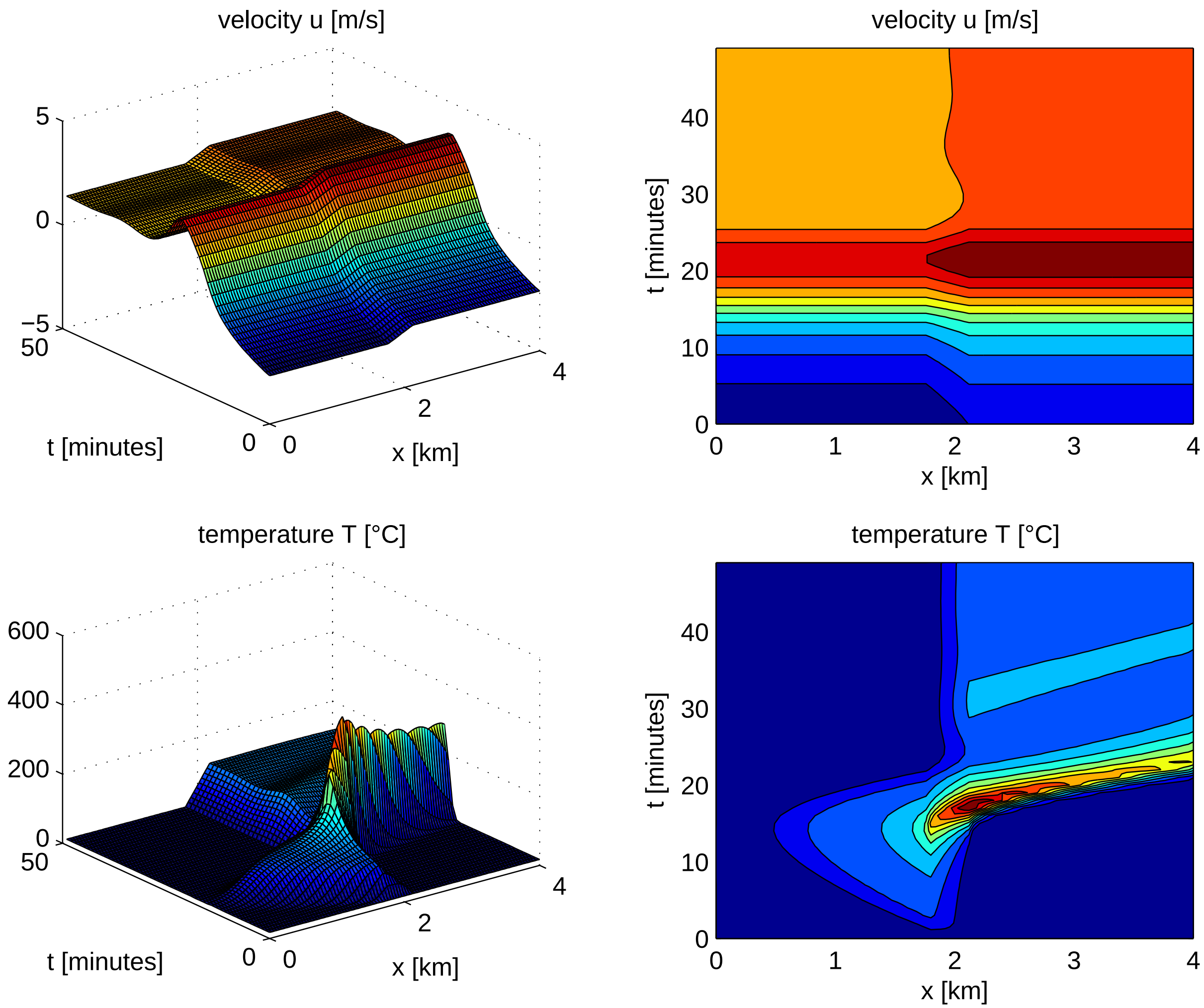


Example 1: Model tunnel

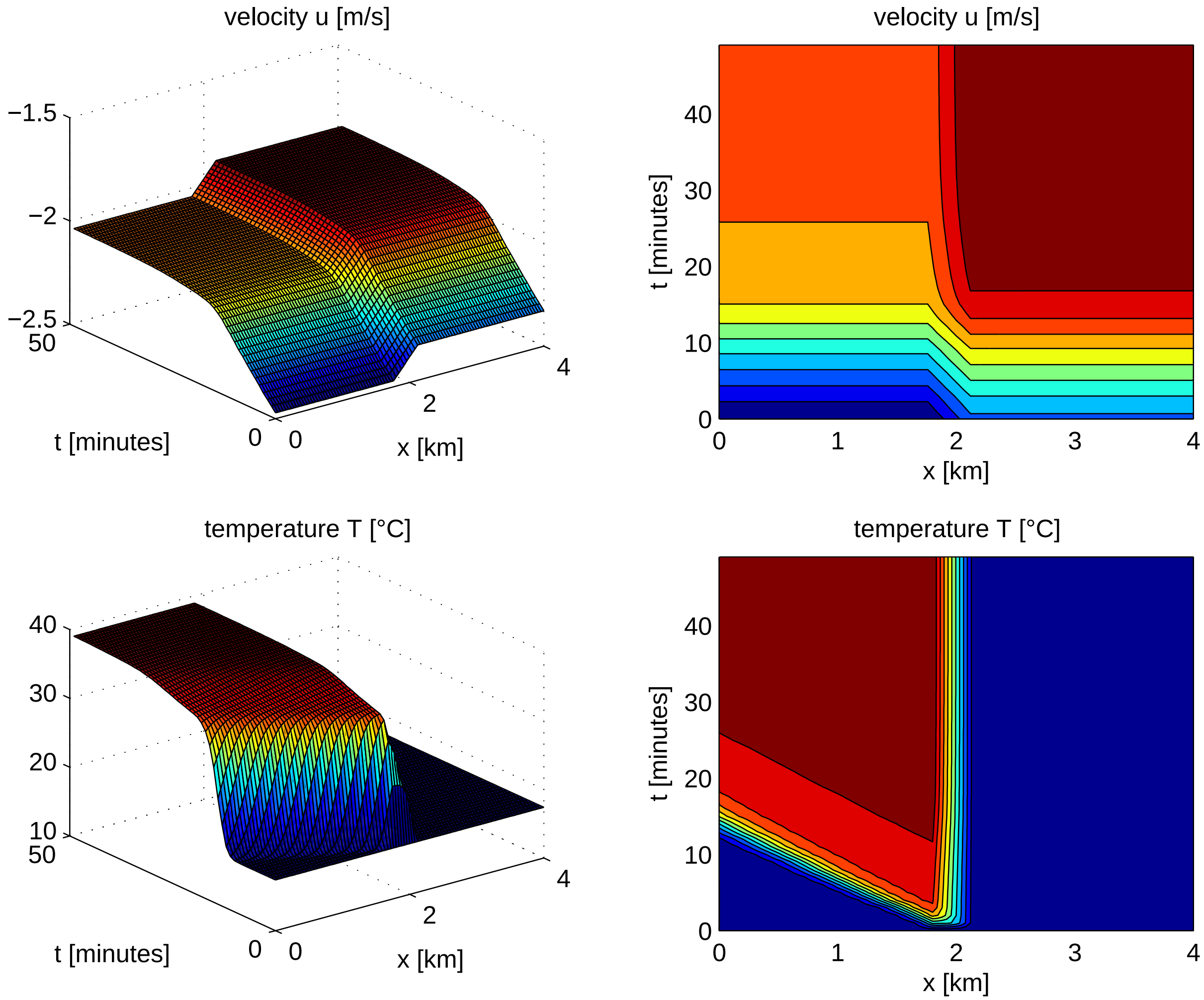
Length	4 km
cross-section	100 m^2
slope	3 %
pressure loss coefficient ξ	0.1
pressure loss $p(L) - p(0)$	12.75 $mbar$
altitude difference	14.4 $mbar$
pressure difference (altitude corrected)	-1.65 $mbar$
mean initial velocity	-2.4 ms^{-1}
heat source	5/20 MW

Experiment: 28. April 2001, Brenner–highway A22

Model tunnel 20 MW



Model tunnel 5 MW

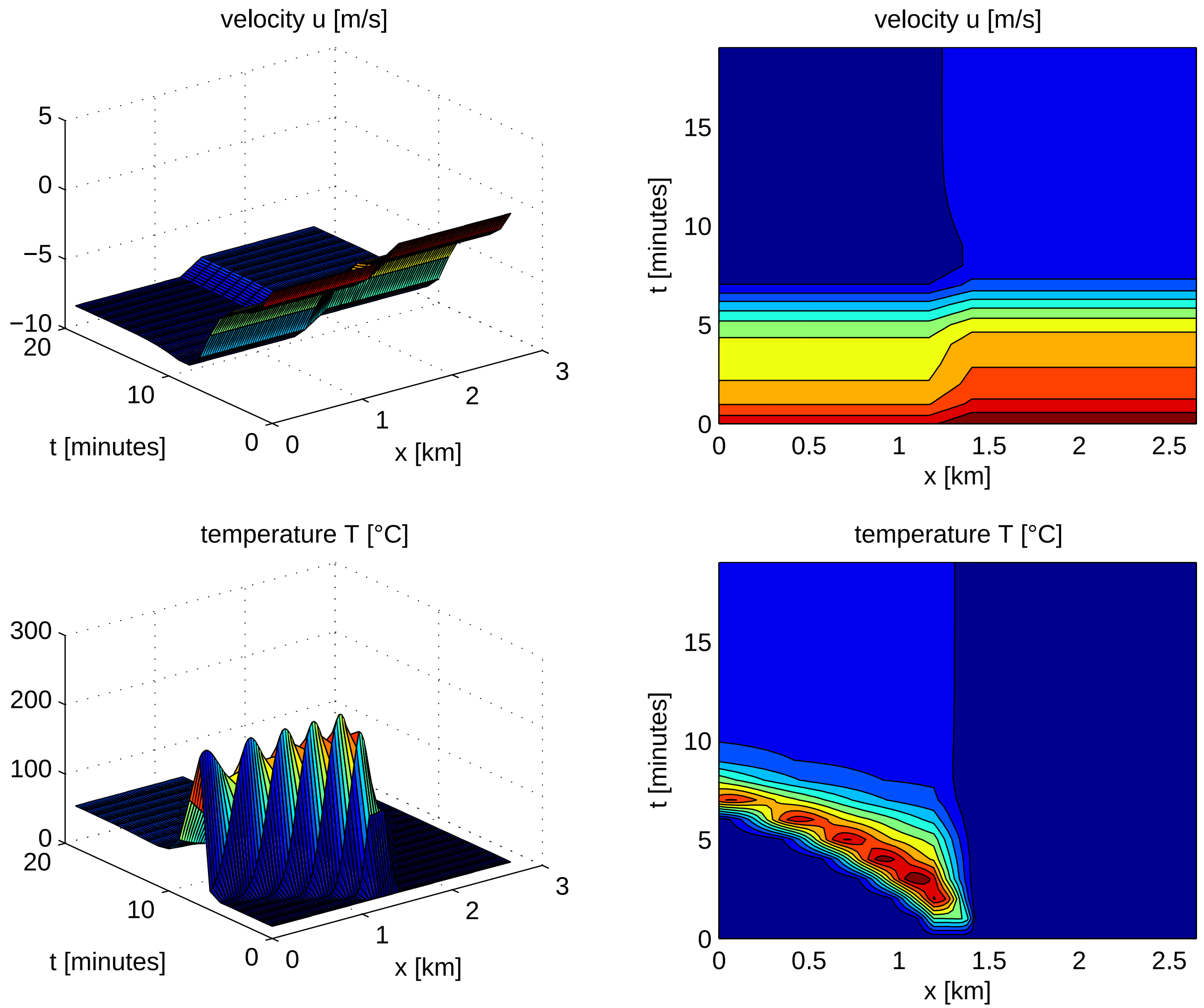


Example 2: Elbtunnel (Hamburg)

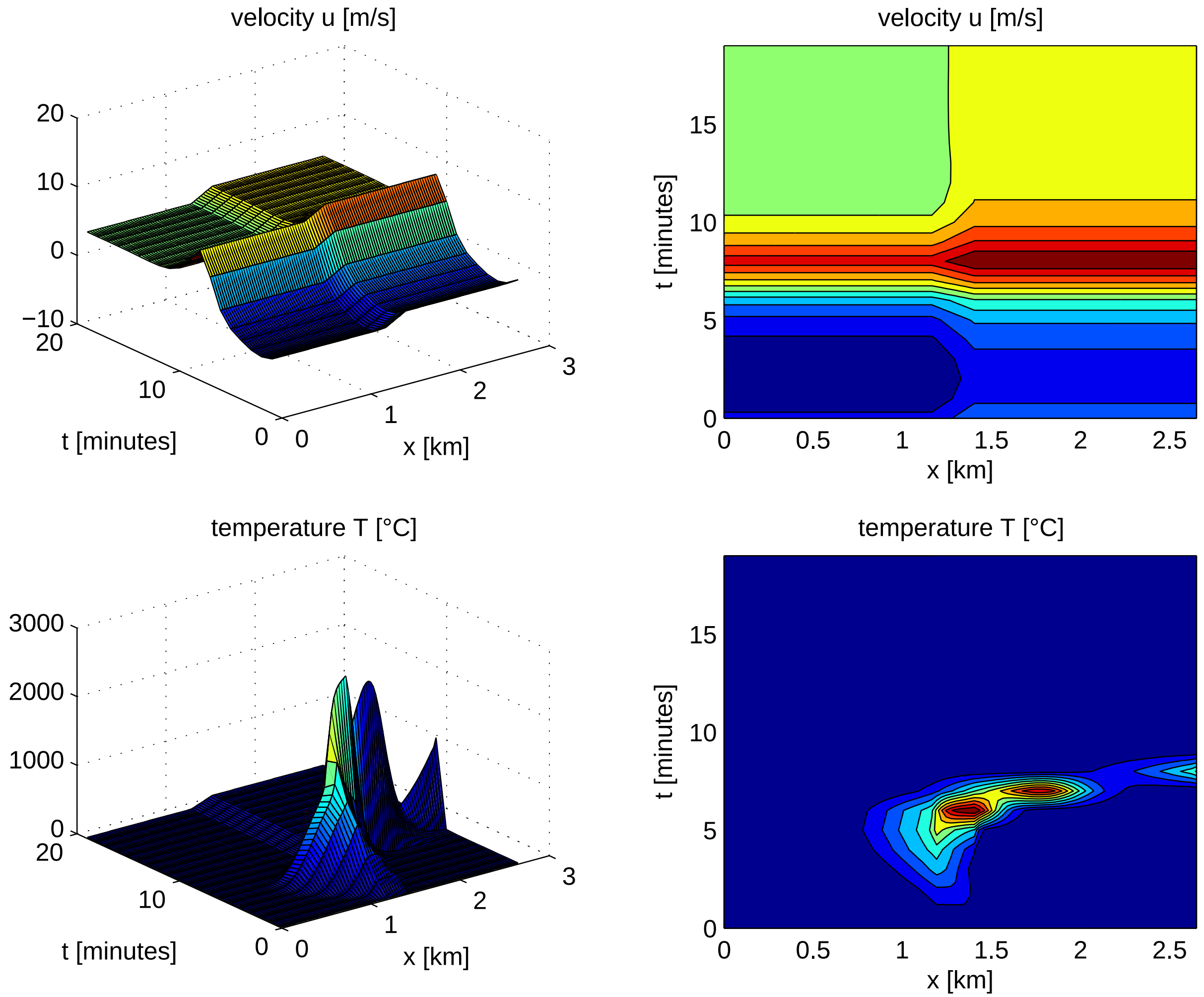
Längth	2.65 km
Crosssection	41 m^2
Slope $\alpha(x)$	$\begin{cases} -3.5\,\%, & 0 \leq x < 0.6km \\ -3.5\,\% & +20.3(x-0.6)\,\%k \\ & 0.6 \leq x < 0.9km \\ 2.6\,\%, & 0.9km \leq x \leq 2.65k \end{cases}$
Pressure loss coefficient ξ	0.007
Pressure diff. $p(L) - p(0)$	1.89 $mbar$
Pressure diff. (due to difference in altitude)	2.59 $mbar$
Pressure diff.(altitude corrected)	-0.7 $mbar$
Mean initial velocity	0 ms^{-1}
Heatsource	15/25 MW

Experiment with Turbolöscher 1999

Elbtunnel 15 MW



Elbtunnel 35 MW



Tunnelnetworks: (I.G., M. Kraft '04)

ventilation exits, ventilation systems, Tunnel bifurcatios etc.

Model: Graph: tunnel-pieces are the edges, bifurcations are the knodes

On the edges: one-dimensional Model for ρ, u, p.

In the knodes: mass-, momentum-, energy conservation give directly the densities for the "outflowing" tunnels . The velocities for the "outflowing" tunnels and the pressure in the knodes are unknowns. The nodes are easy to handle compared to an compressible approach.

Physically motivated monotonicity property:
increasing the pressure in a knode increases the outflow and decreases the inflow.

For explicit numerical methods in every time-step we solve a linear system for the pressures in the knodes.

Outlook

- Modeling:

  variable cross-sections
  big slopes
  radiation
  sprinkling systems
  higher dimensions in regions of special interest

- Analysis:

  stability
  small Mach number limit
  initial time layer problem

- Numerics:

  adaptive scheme