Edge detection within optical flow by multidimensional control

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Problem formulation:

- *Find to a given image sequence* $\{\, I(s,t)\,\}$, $0 \leqslant t \leqslant T$, *a field of displacements* $(\, X_1(s,t)\,,\, X_2(s,t)\,)$, *which leaves the greyscale values invariant:*

$$I(s_1, s_2, t) \,=\, I(\, s_1 - X_1(s,t)\,,\, s_2 - X_2(s,t)\,,\, 0\,)\,, \quad 0 \leqslant t \leqslant T\,,$$

$$X_1(s,0) \,=\, 0\,,\;\; \mathrm{X}_2(s,0) \,=\, 0\,.$$

- *The optical flow* $\big(\,(X_1)_t(s,t)\,,\,(X_2)_t(s,t)\big) \,=\, \big(\,x_1(s,t)\,,\,x_2(s,t)\big)$ *is the associated velocity field:*

$$I_{s_1}(s,0)\cdot(X_1)_t(s,0) \,+\, I_{s_2}(s,0)\cdot(X_2)_t(s,0) \,+\, I_t(s,0) \,=\, 0\,.$$

Problem formulation:

- *Find to a given image sequence* $\{ I(s,t) \}$, $0 \leqslant t \leqslant T$, *a field of displacements* $(X_1(s,t) , X_2(s,t))$, *which leaves the greyscale values invariant:*

$$I(s_1, s_2, t) = I(s_1 - X_1(s,t) , s_2 - X_2(s,t) , 0) , \quad 0 \leqslant t \leqslant T ,$$
$$X_1(s,0) = 0 , \ \mathrm{X}_2(s,0) = 0 .$$

- *The optical flow* $\big((X_1)_t(s,t) , (X_2)_t(s,t) \big) = \big(x_1(s,t) , x_2(s,t) \big)$ *is the associated velocity field:*

$$I_{s_1}(s,0) \cdot x_1(s,0) + I_{s_2}(s,0) \cdot x_2(s,0) + I_t(s,0) = 0 .$$

Problem formulation:

• *Find to a given image sequence* $\{ I(s,t) \}$, $0 \leqslant t \leqslant T$, *a field of displacements* $(X_1(s,t), X_2(s,t))$, *which leaves the greyscale values invariant:*

$$I(s_1, s_2, t) = I(s_1 - X_1(s,t), s_2 - X_2(s,t), 0), \quad 0 \leqslant t \leqslant T,$$
$$X_1(s,0) = 0, \ \mathrm{X}_2(s,0) = 0.$$

• *The optical flow* $\big((X_1)_t(s,t), (X_2)_t(s,t) \big) = \big(x_1(s,t), x_2(s,t) \big)$ *is the associated velocity field:*

$$I_{s_1}(s,0) \cdot x_1(s,0) + I_{s_2}(s,0) \cdot x_2(s,0) + I_t(s,0) = 0.$$

Numerous applications:

• *Video data compression:* [Hinterberger/Scherzer 2001]

• *Automatic retouching of movie sequences:* [Grossauer 2006]

• *Depth from stereo:* [Slesareva/Bruhn/Weickert 2005]

• *Extraction of object and motion informations in traffic monitoring, robotics, medicine:* [Brox/Bruhn/Weickert 2006], [Talukder/Matthies 2004], [Kumar et al. 2001], ...

- *For a given pair of images* $I(s,t_N)$, $I(s,t_{N+1})$ *with* $\Delta t = t_{N+1} - t_N = 1$, *the optical flow constraint reads with reference to* t_N *as*

$$I_{s_1}(s,t_N)\,x_1(s) \,+\, I_{s_2}(s,t_N)\,x_2(s) \,+\, I_t(s,t_N) \,=\, 0 \quad \forall\, s \in \Omega\,.$$

- *Regularization by variational methods:*

$$(\mathrm{V})_1: \quad F(x_1,x_2) \,=\, \int_\Omega \Big(I_{s_1}(s)\,x_1(s) \,+\, I_{s_2}(s)\,x_2(s) \,+\, I_t(s) \Big)^2 \,ds$$
$$+\,\mu\cdot\int_\Omega r(s,\nabla x_1(s),\nabla x_2(s))\,ds \longrightarrow \textit{inf!}; \quad (x_1,x_2) \in W_0^{1,p}(\Omega,\mathbb{R}^2)$$

with $1 \leqslant p < \infty$, $I \in W_0^{1,\infty}(\Omega \,\times\, (\,t_N - \delta\,,\, t_{N+1} + \delta\,)\,,\,\mathbb{R})$, $\mu > 0$ *and* $r \in C^2(\Omega \,\times\, \mathbb{R}^4\,,\,\mathbb{R})$.

- *The objective consists of defect minimization term and regularization term with regularization parameter* μ.
- *The gradients are related to* s_1 *and* s_2 *only; the dependence on* $t = t_N$ *has been dropped in notation.*

First possibility: Variational problem with Ambrosio-Tortorelli functional

$$
\begin{aligned}
(\mathrm{V})_1: \quad F(x_1, x_2) \,=\, & \int_\Omega \Big(I_{s_1}(s)\, x_1(s) \,+\, I_{s_2}(s)\, x_2(s) \,+\, I_t(s) \Big)^2 ds \\
& + \mu \cdot \int_\Omega r(s, \nabla x_1(s), \nabla x_2(s))\, ds \longrightarrow \mathit{inf!}; \quad (x_1, x_2) \in W_0^{1,p}(\Omega, \mathbb{R}^2)
\end{aligned}
$$

First possibility: Variational problem with Ambrosio-Tortorelli functional

$$
\begin{aligned}
(\mathrm{V})_2: \quad F(x_1, x_2, k) \;=\; & c_1(\varepsilon) \int_\Omega \Big(I_{s_1}(s)\, x_1(s) \,+\, I_{s_2}(s)\, x_2(s) \,+\, I_t(s) \Big)^2 ds \\
& + \, c_2(\varepsilon) \int_\Omega \Big(\big|\, \nabla x_1(s) \,\big|^2 + \big|\, \nabla x_2(s) \,\big|^2 \Big) \cdot \Big(k(s)^2 + c_4(\varepsilon) \Big)\, ds \\
& + c_3(\varepsilon) \int_\Omega \Big(\varepsilon \cdot \big|\, \nabla k(s) \,\big|^2 + \frac{1}{4\,\varepsilon} \big(1 - k(s) \big)^2 \Big)\, ds \longrightarrow \textit{inf!}; \quad (x_1, x_2, k) \in W_0^{1,2}(\Omega, \mathbb{R}^2) \times W_0^{1,2}(\Omega, \mathbb{R})
\end{aligned}
$$

- $k(s) \approx 0$ *if* $s \in \Omega$ *belongs to an edge within* (x_1, x_2), *else* $k(s) \approx 1$.

First possibility: Variational problem with Ambrosio-Tortorelli functional

$$\begin{aligned}(\mathrm{V})_2: \quad F(x_1,x_2,k) \;=\; & c_1(\varepsilon)\int_\Omega \Big(I_{s_1}(s)\,x_1(s) \,+\, I_{s_2}(s)\,x_2(s) \,+\, I_t(s)\Big)^2\,ds \\ & +\, c_2(\varepsilon)\int_\Omega \Big(\big|\,\nabla x_1(s)\,\big|^2 + \big|\,\nabla x_2(s)\,\big|^2\Big)\cdot\Big(k(s)^2 + c_4(\varepsilon)\Big)\,ds \\ & + c_3(\varepsilon)\int_\Omega \Big(\varepsilon\cdot\big|\,\nabla k(s)\,\big|^2 + \frac{1}{4\,\varepsilon}\,\big(1-k(s)\,\big)^2\Big)\,ds \longrightarrow \textit{inf!}; \quad (x_1,x_2,k)\in W_0^{1,2}(\Omega,\mathbb{R}^2)\,\times\,W_0^{1,2}(\Omega,\mathbb{R})\end{aligned}$$

- $k(s)\approx 0$ *if* $s\in\Omega$ *belongs to an edge within* (x_1,x_2), *else* $k(s)\approx 1$.

Second possibility: Optimal control reformulation

Add to $(\mathrm{V})_1$ *a convex constraint for* ∇x_1 *and* ∇x_2*:*

$$\begin{aligned}(\mathrm{V})_1: \quad F(x_1,x_2) \;=\; & \int_\Omega \Big(I_{s_1}(s)\,x_1(s) \,+\, I_{s_2}(s)\,x_2(s) \,+\, I_t(s)\Big)^2\,ds \\ & +\mu\cdot\int_\Omega r(s,\nabla x_1(s),\nabla x_2(s))\,ds \longrightarrow \textit{inf!}; \quad (x_1,x_2)\in W_0^{1,p}(\Omega,\mathbb{R}^2)\end{aligned}$$

First possibility: Variational problem with Ambrosio-Tortorelli functional

$$\begin{aligned}(\mathrm{V})_2:\quad F(x_1,x_2,k) \;=\; & c_1(\varepsilon)\int_\Omega \Big(I_{s_1}(s)\,x_1(s) \,+\, I_{s_2}(s)\,x_2(s) \,+\, I_t(s)\Big)^2\,ds \\ & + c_2(\varepsilon)\int_\Omega \Big(\big|\,\nabla x_1(s)\,\big|^2 + \big|\,\nabla x_2(s)\,\big|^2\Big)\cdot\Big(k(s)^2 + c_4(\varepsilon)\Big)\,ds \\ & + c_3(\varepsilon)\int_\Omega \Big(\varepsilon\cdot\big|\,\nabla k(s)\,\big|^2 + \frac{1}{4\,\varepsilon}\,\big(1-k(s)\,\big)^2\Big)\,ds \longrightarrow \mathit{inf!};\quad (x_1,x_2,k)\in W_0^{1,2}(\Omega,\mathbb{R}^2)\,\times\,W_0^{1,2}(\Omega,\mathbb{R})\end{aligned}$$

- $k(s)\approx 0$ *if* $s\in\Omega$ *belongs to an edge within* (x_1,x_2)*, else* $k(s)\approx 1$.

Second possibility: Optimal control reformulation

$$\begin{aligned}(\mathrm{P})_1:\quad F(x,u) = & \int_\Omega \Big(I_{s_1}(s)\,x_1(s) + I_{s_2}(s)\,x_2(s) + I_t(s)\Big)^2\,ds \\ & + \mu\cdot\int_\Omega r(s,u_{11}(s),u_{12}(s),u_{21}(s),u_{22}(s))\,ds \;\longrightarrow\; \mathit{inf!};\quad (x,u)\in W_0^{1,p}(\Omega,\mathbb{R}^2)\,\times\,L^\infty(\Omega,\mathbb{R}^4)\,;\end{aligned}$$

$$Jx(s) \;=\; u(s)\quad (\forall)\,s\in\Omega\,;$$

$$u\;\in\;\mathrm{U}\;=\;\Big\{\,u\in L^p(\Omega,\mathbb{R}^4)\;\Big|\;\big|\,u_{11}(s)\,\big|^q + \big|\,u_{12}(s)\,\big|^q + \big|\,u_{21}(s)\,\big|^q + \big|\,u_{22}(s)\,\big|^q \;\leqslant\; R^q\;\;(\forall)\,s\in\Omega\,\Big\}\,.$$

- *Interpret as "edges" those subsets of* Ω *where the control restriction is (nearly) active.*

Problem formulation:

$$(\mathrm{P})_0: \quad F(x,u) = \int_\Omega f(s, x(s), u(s))\, ds \longrightarrow \textit{inf!}; \quad (x,u) \in W_0^{1,p}(\Omega, \mathbb{R}^n) \times L^p(\Omega, \mathbb{R}^{nm});$$

$$Jx(s) = \left(\frac{\partial x_i}{\partial s_j}(s) \right)_{i,j} = \Big(u_{ij}(s) \Big)_{i,j} \quad (\forall)\, s \in \Omega;$$

$$u \in \mathrm{U} = \left\{ u \in L^p(\Omega, \mathbb{R}^{nm}) \;\middle|\; u(s) \in \mathrm{K} \;\; (\forall)\, s \in \Omega \right\}$$

Analytical framework:

- *Dimensions:* $\Omega \subset \mathbb{R}^m$, $n \geqslant 1$, $m \geqslant 2$.
- *Integrand:* $f(s, \xi, v): \Omega \times \mathbb{R}^n \times \mathbb{R}^{nm} \to \mathbb{R}$ — L^∞ *in* s, *and* C^1 *in all* ξ_i *and* v_{ij}.
- *Control restriction:* $\mathrm{K} \subset \mathbb{R}^{nm}$ — *convex body with* $\mathfrak{o} \in \operatorname{int}(\mathrm{K})$.

- **Existence of global minimizers:**

If $f(s,\xi,v)$ *is convex in the last argument* v *and obeys a growth condition of the type*

$$\big| f(s,\xi,v) \big| \leqslant \varphi_1(s) + \varphi_2(|\xi|, |v|) \quad (\forall)\, s \in \Omega \quad \forall\,(\xi,v) \in \mathbb{R}^n \times \mathrm{K}$$

with $\varphi_1 \in L^1(\Omega,\mathbb{R})$, $\varphi_1(s) \geqslant 0$ $(\forall)\, s \in \Omega$, *and* $\varphi_2 \in C^0(\mathbb{R}^n \times \mathrm{K}\,,\mathbb{R})$, $\varphi_2(|\xi|, |\,, v|) \geqslant 0$ $\forall\,(\xi,v) \in \mathbb{R}^n \times \mathrm{K}$ *where* φ_2 *is monotonically increasing in* $|\xi|$ *as well as in* $|v|$ *then* $(\mathrm{P})_0$ *admits a global minimizer.*

- **Properties of minimizing sequences:**

They contain subsequences $\{(x^N, u^N)\}$ *, which converge to a global minimizer uniformly in* x *and weakly** *in* u*.*

- **Important consequence:**

Application of direct methods to convex problems of the type $(\mathrm{P})_0$ *is in principle justified.*

- **We first discretize, then optimize:**

— *Domain:* Ω *is represented by* $(K \times L)$ *pixels with edge length* 1.

— *Variables:* $2KL$ *state variables* $x_{kl}^{(1)}$, $x_{kl}^{(2)}$ *and* $4KL$ *control variables* $u_{kl}^{(11)}$, ... , $u_{kl}^{(22)}$.

— *Objective: Discretization by trapezium rule.*

— *Derivatives: forward Euler differences with* $h = \Delta t = 1$

$$\begin{pmatrix} u_{kl}^{(11)} & u_{kl}^{(12)} \\ u_{kl}^{(21)} & u_{kl}^{(22)} \end{pmatrix} = \begin{pmatrix} x_{k+1,l}^{(1)} - x_{k,l}^{(1)} & x_{k,l+1}^{(1)} - x_{k,l}^{(1)} \\ x_{k+1,l}^{(2)} - x_{k,l}^{(2)} & x_{k,l+1}^{(2)} - x_{k,l}^{(2)} \end{pmatrix} \quad \forall\, k,\, l\,.$$

— *Control restriction:*

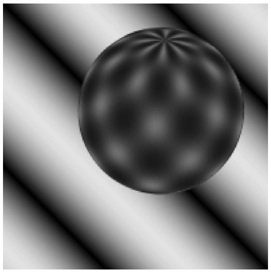
$$0 \leqslant \left| u_{kl}^{(11)} \right|^q + \left| u_{kl}^{(12)} \right|^q + \left| u_{kl}^{(21)} \right|^q + \left| u_{kl}^{(22)} \right|^q \leqslant R^q \quad \forall\, k,\, l\,.$$

- **Solving the discretized problems**

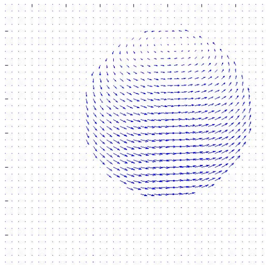
by interior-point methods (solvers IPOPT, LOQO), embedded in an AMPL interface; image output via MATLAB.

Image data and visualization of the results

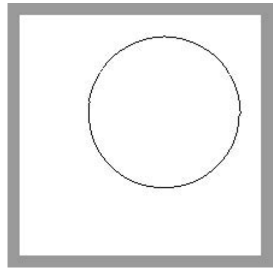
Rotating Sphere Sequence: *(taken from* [McCane/Novins/Crannitch/Galvin 2001] *)*



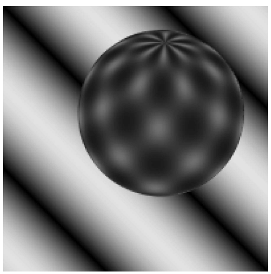
- *Image # 13*



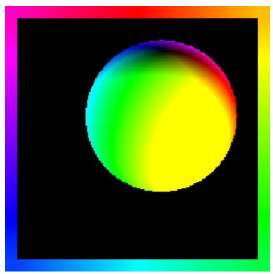
- *Ground-truth, vector plot*



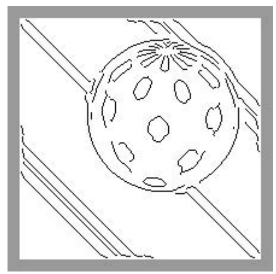
- *Motion edge sketch*



- *Image # 14*



- *Colorful orientation plot*



- *Intensity edge sketch*

Deviation from the ground-truth: AAE

$$AAE(\hat{x},x) \;=\; \sum_{k=1}^{K}\sum_{l=1}^{L} \arccos\Bigg(\frac{\hat{x}_1(s_{kl})\,x_1(s_{kl}) + \hat{x}_2(s_{kl})\,x_2(s_{kl}) + 1}{\sqrt{(\hat{x}_1(s_{kl}))^2 + (\hat{x}_2(s_{kl}))^2 + 1}\;\sqrt{(x_1(s_{kl}))^2 + (x_2(s_{kl}))^2 + 1}} \Bigg)\,.$$

Detection of motion edges: MEE

$$MEE(\hat{x},x) \;=\; \sum_{k=1}^{K}\sum_{l=1}^{L} \big(k_m(s_{kl}) - k(s_{kl}) \big)^2 \;\approx\; \int_\Omega \big(k_m(s) - k(s) \big)^2\, ds\,.$$

- *k_m is based on the modulus $|\,\hat{x}\,|$ of the ground-truth.*

Detection of intensity edges: IEE

$$IEE(\hat{x},x) \;=\; \sum_{k=1}^{K}\sum_{l=1}^{L} \frac{1}{2}\Big(\big(k_g^{(N)}(s_{kl}) - k(s_{kl}) \big)^2 \;+\; \big(k_g^{(N+1)}(s_{kl}) - k(s_{kl}) \big)^2 \Big)$$

$$\approx \int_\Omega \frac{1}{2}\Big(\big(k_g^{(N)}(s) - k(s) \big)^2 \;+\; \big(k_g^{(N+1)}(s) - k(s) \big)^2 \Big)\, ds\,.$$

- *$k_g^{(N)}$, $k_g^{(N+1)}$ are based on the images $I(s,t_N)$, $I(s,t_{N+1})$.*

Control problem:

$$(\mathrm{P})_2: \; F(x,u) = \int_\Omega \Big(I_{s_1}(s)\, x_1(s) + I_{s_2}(s)\, x_2(s) + I_t(s) \Big)^2 ds \longrightarrow \textit{inf!}; \; (x,u) \in W_0^{1,\infty}(\Omega, \mathbb{R}^2) \times L^\infty(\Omega, \mathbb{R}^4);$$

$$Jx(s) = \begin{pmatrix} u_{11}(s) & u_{12}(s) \\ u_{21}(s) & u_{22}(s) \end{pmatrix} \quad (\forall)\, s \in \Omega;$$

$$\big| u_{11}(s) \big|^q + \big| u_{12}(s) \big|^q + \big| u_{21}(s) \big|^q + \big| u_{22}(s) \big|^q \leqslant R^q \quad (\forall)\, s \in \Omega;$$

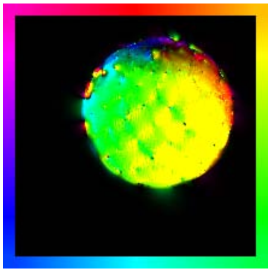
Edge detector:

$$k(s) = 1 - \frac{\big| u_{11}(s) \big|^q + \big| u_{12}(s) \big|^q + \big| u_{21}(s) \big|^q + \big| u_{22}(s) \big|^q}{\underset{s \in \Omega}{\mathrm{Max}} \Big(\big| u_{11}(s) \big|^q + \big| u_{12}(s) \big|^q + \big| u_{21}(s) \big|^q + \big| u_{22}(s) \big|^q \Big)}.$$

Parameters:

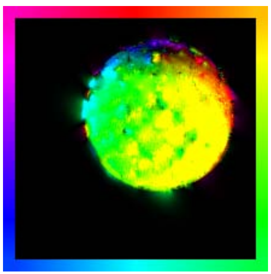
$q \geqslant 1$ *and* $R > 0$ *(kind and sharpness of the control restriction)*

Selected results: *(variation in q and R)*



$q = 2,\ R = 4$

$AAE = 4.11,$
$MEE = 2.16,\ IEE = 4.40$



$q = 2,\ R = 0.5$

$AAE = 4.22,$
$MEE = 7.25,\ IEE = 7.89$

Control problem:

$$(\mathrm{P})_3: \quad F(x,u) = \int_\Omega \Big(\big(I_{s_1}(s)\, x_1(s) + I_{s_2}(s)\, x_2(s) + I_t(s) \big)^2 + \varepsilon \Big)^{1/2} ds + \mu \cdot \int_\Omega \Big(\big| u_{11}(s) \big|^p + \big| u_{12}(s) \big|^p + \big| u_{21}(s) \big|^p + \big| u_{22}(s) \big|^p + \varepsilon \Big)^{1/p} ds \longrightarrow \textit{inf!}; \quad (x,u) \in W_0^{1,\infty}(\Omega, \mathbb{R}^2) \times L^\infty(\Omega, \mathbb{R}^4);$$

$$Jx(s) = \begin{pmatrix} u_{11}(s) & u_{12}(s) \\ u_{21}(s) & u_{22}(s) \end{pmatrix} \quad (\forall)\, s \in \Omega;$$

$$\big| u_{11}(s) \big|^q + \big| u_{12}(s) \big|^q + \big| u_{21}(s) \big|^q + \big| u_{22}(s) \big|^q \leqslant R^q \quad (\forall)\, s \in \Omega;$$

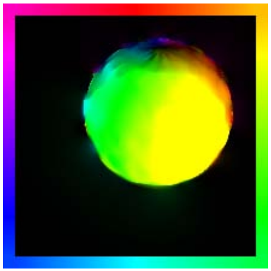
Edge detector:

$$k(s) = \frac{\kappa(s)}{\underset{s \in \Omega}{\mathrm{Max}}\ \kappa(s)} \cdot \begin{cases} 1 \mid \kappa(s) \geqslant \alpha\, R^q; \\ 0 \mid \kappa(s) < \alpha\, R^q \end{cases} \quad \textit{with } \kappa(s) = \big| u_{11}(s) \big|^q + \big| u_{12}(s) \big|^q + \big| u_{21}(s) \big|^q + \big| u_{22}(s) \big|^q.$$

Parameters:

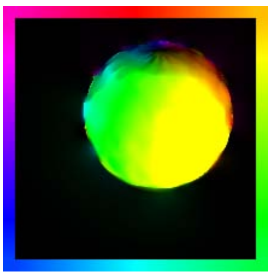
$\varepsilon > 0$ (robustness), $p > 1$ and $\mu > 0$ (regularization), $q \geqslant 1$ and $R > 0$ (kind and sharpness of the control restriction), $\alpha \geqslant 0$ (threshold for edge output)

Selected results: *(variation in R)*



$\varepsilon = 0.001,\ \mu = 0.001,\ p = 2,$
$q = 2,\ R = 4,\ \alpha = 0$

$AAE = 3.08,$
$MEE = 1.89,\ IEE = 4.28$



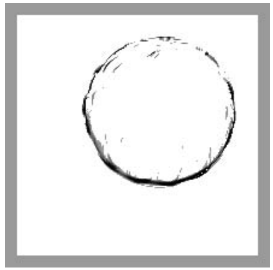
$\varepsilon = 0.001,\ \mu = 0.001,\ p = 2,$
$q = 2,\ R = 2,\ \alpha = 0$

$AAE = 3.07,$
$MEE = 1.89,\ IEE = 4.28$

Numerical results (II): adaptation of the edge detector

$\varepsilon = 0.001$, $\mu = 0.001$, $p = 2$,
$q = 2$, $R = 0.5$, $\alpha = 0$

$AAE = 3.23$,
$MEE = 3.03$, $IEE = 4.72$



$\varepsilon = 0.001$, $\mu = 0.001$, $p = 2$,
$q = 2$, $R = 0.5$, $\alpha = 0.1$

$AAE = 3.23$,
$MEE = 2.87$, $IEE = 4.78$

$\varepsilon = 0.001$, $\mu = 0.001$, $p = 2$,
$q = 2$, $R = 0.5$, $\alpha = 0.2$

$AAE = 3.23$,
$MEE = 2.82$, $IEE = 4.78$

Control problem: *(with decomposition of control variables into positive and negative part)*

$$(\mathrm{P})_4: \quad F(x,u) = \int_\Omega \Big(\big(I_{s_1}(s)\,x_1(s) + I_{s_2}(s)\,x_2(s) + I_t(s) \big)^2 + \varepsilon \Big)^{1/2} ds$$

$$+ \mu \cdot \int_\Omega \sum_{i,j=1}^{2} \big(u_{ij}^+(s) + u_{ij}^-(s) \big)\, ds \longrightarrow \textit{inf!}; \quad (x,u) \in W_0^{1,\infty}(\Omega,\mathbb{R}^2) \times L^\infty(\Omega,\mathbb{R}^8)\,;$$

$$Jx(s) = \begin{pmatrix} u_{11}^+(s) - u_{11}^-(s) & u_{12}^+(s) - u_{12}^-(s) \\ u_{21}^+(s) - u_{21}^-(s) & u_{22}^+(s) - u_{22}^-(s) \end{pmatrix} \quad (\forall)\, s \in \Omega\,;$$

$$\sum_{i,j=1}^{2} \big(u_{ij}^+(s) + u_{ij}^-(s) \big) \leqslant R \quad (\forall)\, s \in \Omega\,;$$

$$u_{ij}^+(s),\ u_{ij}^-(s) \geqslant 0 \quad (\forall)\, s \in \Omega,\ 1 \leqslant i,\, j \leqslant 2\,.$$

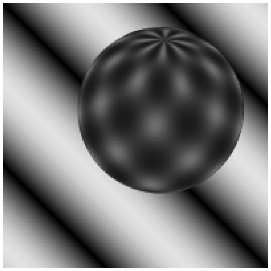
Edge detector:

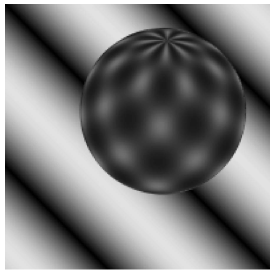
$$k(s) = 1 - \frac{\sum\limits_{i,j=1}^{2} \big(u_{ij}^+(s) + u_{ij}^-(s) \big)}{\mathop{\mathrm{Max}}\limits_{s \in \Omega} \sum\limits_{i,j=1}^{2} \big(u_{ij}^+(s) + u_{ij}^-(s) \big)}\,.$$

Parameters:

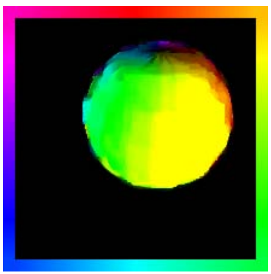
$\varepsilon > 0$ *(robustness)*, $\mu > 0$ *(regularization)*, $R > 0$ *(sharpness of the control restriction)*

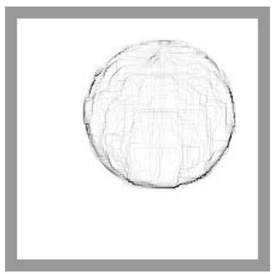
Selected results: Rotating Sphere Sequence





- *Image # 13*
- *Image # 14*

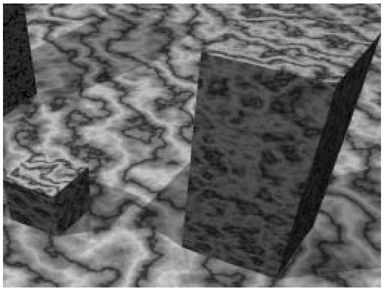




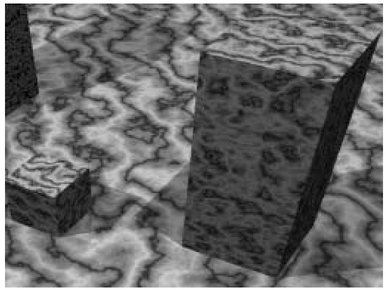
$\varepsilon = 0.001,\ \mu = 0.002,\ R = 2$

$AAE = 2.57,$
$MEE = 2.11,\ IEE = 4.38$

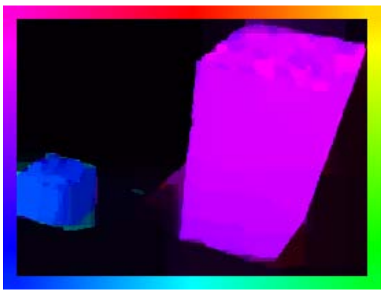
Selected results: New Marbled Block Sequence

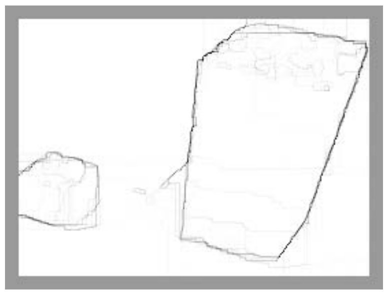


- *Part of Image #163*



- *Part of Image #164*





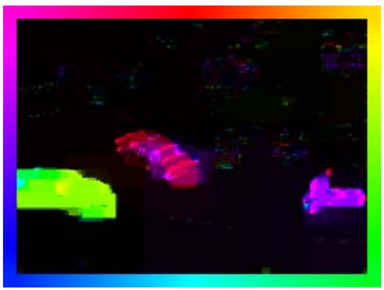
$\varepsilon = 0.001$, $\mu = 0.07$, $R = 2$

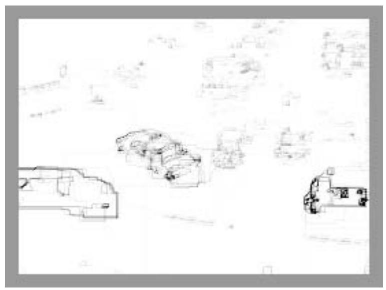
$AAE = 6.44$,
$MEE = 2.87$, $IEE = 8.84$

Selected results: Hamburg Taxi Sequence

- *Image #22*

- *Image #23*





$\varepsilon = 0.001, \ \mu = 0.01, \ R = 2$

AAE, MEE, IEE *not available (sequence without ground-truth)*

Conclusions

- **Numerical methods for convex Dieudonné-Rashevsky type problems ...**

— *have been established for the first time.*

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- **The control restriction ...**

— *acts itself as an* L^∞*-regularization.*

— *can amplify or modify the effects of the known regularization terms.*

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— *have been established for the first time.*

- **The control restriction ...**

— *acts itself as an* L^∞*-regularization.*

— *can amplify or modify the effects of the known regularization terms.*

- **Simultaneous edge detection via optimal control ...**

— *presents an interesting alternative to the Ambrosio-Tortorelli variational problem, with comparable accuracy.*

— *allows a simple adjustment of the edge output.*

— *provides a primal approach for problems with TV regularization.*

- **Variational methods for determining optical flow:**

1. [AUBERT/KORNPROBST 2006] *Aubert, G.; Kornprobst, P.: Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations. Springer; New York etc. 2006, 2nd ed., pp. 250 – 261*
2. [BRUHN/WEICKERT/KOHLBERGER/SCHNÖRR 2006] *Bruhn, A.; Weickert, J.; Kohlberger, T.; Schnörr, C.: A multigrid platform for real-time motion computation with discontinuity-preserving variational methods. Int. J. Computer Vision* **70** *(2006), 257 – 277*

- **Control problems in image processing:**

3. [WAGNER 2007A] *Wagner, M.: Pontryagin's maximum principle for multidimensional control problems in image processing. BTU Cottbus, Preprint-Reihe Mathematik M-10/2007. To appear in: J. Optim. Theory Appl.*
4. [WAGNER 2007B] *Wagner, M.: On the lower semicontinuous quasiconvex envelope for unbounded integrands (I). BTU Cottbus, Preprint-Reihe Mathematik M-04/2006. To appear in: ESAIM: Control, Optimisation and Calculus of Variations.*
5. [BRUNE/MAURER/WAGNER 2008] *Brune, C.; Maurer, H.; Wagner, M.: Kantenerkennung im optischen Fluß als mehrdimensionales Steuerungsproblem. BTU Cottbus, Preprint-Reihe Mathematik, M-01/2008.*

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