Numerical modelling of shallow flows on rotating plane including topography and magnetodynamic effects

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Many types of flows, not necessarily involving water, can be characterized as shallow water flows. They describe flows of fluids with a free surface under the influence of gravity, where the vertical dimension is much smaller than any typical horizontal scale. Typical examples of shallow water flows are rivers with their flood plains, flows in lakes generated by wind blows as well as propagation of tsunamis in oceans. The goal of this contribution is to present results of numerical modelling of the shallow water equations by higher order finite volume schemes which belong to the class of genuinely multidimensional well-balanced schemes. In particular, we will concentrate on the so-called finite volume evolution Galerkin (FVEG) method, cf. [1]-[4], but comparisons with other suitable schemes will be presented as well.

• In the first part of our contribution we will present a *well-balanced* FVEG schemes for the system of shallow water equations with the source terms modelling the bottom topography and the Coriolis forces. The FVEG schemes couple the finite volume update with the approximate evolution operators which are based on the theory of bicharacteristics. As a result approximate evolution operators, which are used here instead of classical one-dimensional Riemann solvers, take all of the infinitely many directions of wave propagation into account explicitly. In summary, the FVEG scheme is a genuinely multidimensional method that is explicit in time. A special attention need to be payed to approximate source terms, cf. [5]. The aim is to construct such a method that approximates correctly also equilibrium states, i.e. stationary states as well as their small perturbations. In particular we can prove that our method preserves exactly steady stationary states (the so-called lake at rest state) as well as stationary jets in the rotational frame.

• Further application that we are interested in is the *shallow water magnetohydrodynamic (SMHD) equations*. These equations couple the shallow water system with the Maxwell equations for the magnetic field. The SMHD equations are used to model solar tachocline, which is a thin layer of the solar radius that separates the convective zone from the radiative zone in stars. In [6] we have derived the exact and approximate evolution operators for the system of the SMHD equations in two space dimensions. We have studied more deeply

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the approximation of the spatial derivatives in the evolution operator for singular wave modes, for which the wave front concentrates to a point, as well as for non-singular wave modes. We also discuss suitable techniques to clean the discrete divergence of the magnetic field.

References

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