Social Interaction – A Formal Exploration

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Social Interaction – An Example

Another Example

Information in Social Situations

- Success of situations depends upon information of the agents
- Not too little belief
- Not too much belief
- Higher order belief matters

Our Perspective: Logics for Social Interaction

- Qualitative Modelling of Information
- Descriptive: Adequate representation of the situation
- Goal State: Distribution of Information that should be achieved
- Protocols: Achieving a certain type of Information

Information in Interaction – The logic

Fix a set of atomic propositions P and a set of agent At. Define the epistemic language $\mathcal{L}_K$ as:

$$\varphi := p | \varphi \wedge \varphi | \neg \varphi | K_i \varphi : i \in \mathrm{At}$$

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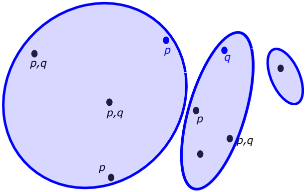
Axioms

- P All propositional validities
- N $K(\varphi \to \psi) \to (K\varphi \to K\psi)$
- T $K\varphi \to \varphi$
- PI $K\varphi \to KK\varphi$
- NI $\neg K\varphi \to K\neg K\varphi$

The Semantics

An epistemic model is a tripel $\langle W, (R_i)_{i \in \mathsf{At}}, V \rangle$ where

- W is a set of worlds
- R_i is an equivalence relation on W
- $V : P \to P(W)$ is an atomic valuation



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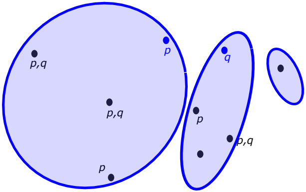

Evaluate the epistemic language on model-world pairs by

- $M, w \vDash p$ iff $w \in V(p)$ $\quad M, w \vDash \neg\varphi$ iff $M, w \nvDash \varphi \ldots$
- $M, w \vDash K_i\psi$ iff for all v with vR_iw: $M, v \vDash \psi$

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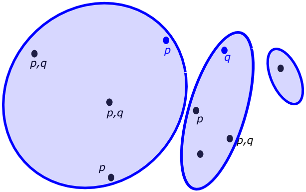

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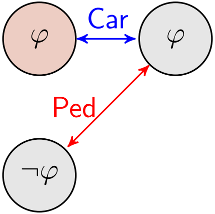
$\mathcal{L}_K$ is sound and complete w.r.t the class of epistemic models

An Example



$\varphi =$ Both approaching at the same time

Information in Interaction – The belief case

Fix a set of atomic propositions P and a set of agent At. Define the doxastic language $\mathcal{L}_B$ as:

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A doxastic model is a tripel $\langle W, (R_i)_{i \in \mathsf{At}}, V \rangle$ where

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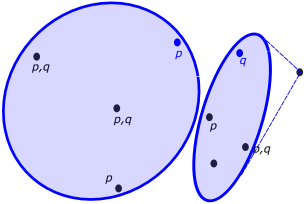

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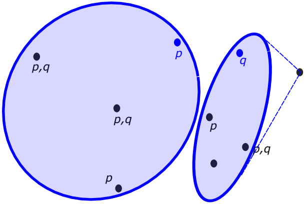

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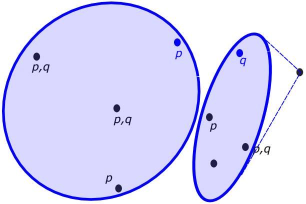

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$\mathcal{L}_B$ is sound and complete w.r.t the class of doxastic models

The Central Question

Which language should we use

- Knowledge: $\mathcal{L}_K$?
- Belief: $\mathcal{L}_B$?
- Knowledge & Belief?
- Common Knowledge?

  Everybody knows φ, Everybody knows everybody knows φ...

- Only Interested in special propositions
- Only fragments of the language?

  Only bounded information. Only *positive belief*...

Some Considerations

- Needs of the situation
- Poor languages can't represent the situation adequately
- Too rich languages might have complexity issues
  - Compactness?
  - (Finite) Realizability?
  - ...

The Questions for Today

- Expressive power
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- Expressive power
  - When does a description language allow to distinguish only few different situations
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- Realizability
  - Can I guarantee that every consistent state description is realizable in a finite model?
- Dynamics
  - How do state descriptions change under information dynamics
  - How to bring about a certain situation?

Let's make things a bit more precise

Let $\mathcal{L}$ be the language with a single atom x

$$\varphi = x|\varphi \wedge \varphi|\neg\varphi|K_i\varphi$$

Definition

A reasoning language is any fragment $\mathcal{L}_{res}$ of $\mathcal{L}$.

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Definition

For a reasoning language $\mathcal{L}_{res}$, a level of $\mathcal{L}_{res}$ information is a set $T \subseteq \mathcal{L}_{res}$ such that the set

$$T \cup \{\neg\varphi|\varphi \in \mathcal{L}_{res} \setminus T\}$$

is consistent.

The first Question:

When does a reasoning language allow for only few (countably many) levels of information?

Why is this a thing

- Take the reasoning language generated by $K_1, K_2, \neg$
  All formulas of the form
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Consider the set

$$\{x, K_1 x, \neg K_2 K_1 x, \neg K_1 \neg K_2 K_1 x, K_2 \neg K_1 \neg K_2 K_1 x\}$$

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$$\begin{aligned}
\neg K_1 x &\to \neg K_2 K_1 x \\
K_1 \neg K_1 x &\to K_1 \neg K_2 K_1 x \\
\neg K_1 x &\to K_1 \neg K_2 K_1 x \text{ Negative Introsp} \\
\neg K_1 \neg K_2 K_1 x &\to K_1 x \text{ Counterpos.} \\
K_2 \neg K_1 \neg K_2 K_1 x &\to K_2 K_1 x
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$$K_2 \neg K_1 \neg K_2 K_1 x \to K_2 K_1 x$$

Not all sets of formulas are consistent

Here is a Central Result

Theorem (Parikh&Krasucki 1992)

Let $\mathcal{L}_K$ be the reasoning language generated by $K_1, \dots K_m$, i.e. the set of all formulas of the form

$$K_1 x,\ K_1 K_2 K_3 K_1 x,\ K_1 K_1 x \dots$$

There are only countably many levels of $\mathcal{L}_K$ information.

The Proof Idea

Let the pre-order $\preceq$ on $\mathcal{L}_K$ formulas be defined by:

$$K_{j_1} \dots K_{j_r} x \preceq K_{i_1} \dots K_{i_m} x$$

iff there is an order preserving embedding from the first to the second formulas, that is, a sequence $s_1 < \dots < s_r$ such that

$$K_{i_{s_l}} = K_{j_l}$$

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Each level of information is downward closed under $\preceq$

- Assume $K_{i_1} \ldots K_{i_r} K_{i_{r+1}} \ldots K_{i_s} \varphi$
- The T axiom implies

$$K_{i_r} K_{i_{r+1}} \ldots K_{i_s} \varphi \rightarrow K_{i_{r+1}} \ldots K_{i_s} \varphi$$

- Thus by normality
$K_{i_1} \ldots K_{i_r} K_{i_{r+1}} \ldots K_{i_s} \varphi \rightarrow K_{i_1} \ldots K_{i_{r+1}} \ldots K_{i_s} \varphi$

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- Every level of knowledge is $\prec$-downward closed
- Hence its complement is uniquely determined by its $\prec$-minimal elements
- But these are an antichain and thus finite
- Hence every level of knowledge is characterized by a countable subset of $\mathcal{L}_K$.

What about belief?

Lemma

The language $\mathcal{L}_B$ generated by $\{B_1, B_2\}$ has uncountably many levels of information.

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- Show that the formulas φ_n defined by

$$\varphi_n := \underbrace{B_1 B_2 B_1 B_2 \dots}_{n \text{ operators}} x$$

  are mutually independent.

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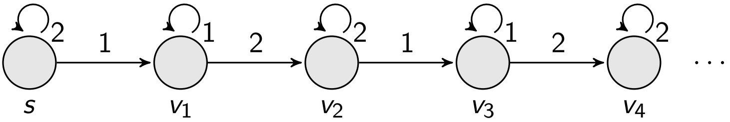

- Lack of T axiom makes all the difference

Back to Knowledge

Let J_i be the knowing whether operator defined as

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Theorem (Hart et al. 96)

Let $\mathcal{L}_J$ be the reasoning language generated by $\{J_1, J_2\}$. Then there are uncountably many levels of $\mathcal{L}_J$-information.

- Again the lack of T makes all the difference
- So where exactly is the fault line among K fragments?

What about judging things possible

Define $L_i\varphi$ as $\neg K_i\neg\varphi$ (φ is compatible with i's information)

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Lemma

Let $\mathcal{L}_L$ be the reasoning language generated by $\{L_1, \ldots, L_n\}$. Then there are at most countably many levels of $\mathcal{L}_L$-information.

- There is a natural bijection between $\mathcal{L}_L$ levels of information and $\mathcal{L}_K$ levels of information.

$$K_{i_1} \ldots K_{i_r} x \leftrightarrow \neg L_{i_1} \ldots L_{i_r} \neg x$$

Lemma

Assume there are at least two agents and let $\mathcal{L}_{L,K}$ be the language generated by $\{L_1, L_2, K_1, K_2\}$. Then there are uncountably many levels of $\mathcal{L}_{L,K}$-information.

Lemma

Assume there are at least two agents and let $\mathcal{L}_{L,K}$ be the language generated by $\{L_1, L_2, K_1, K_2\}$. Then there are uncountably many levels of $\mathcal{L}_{L,K}$-information.

Proof:

- Consider formulas of the form

$$\varphi_n := \underbrace{L_1 L_2 \dots L_1 L_2}_{2^n \ (L_1 L_2) \text{ blocks}} \underbrace{K_1 K_2 \dots K_1 K_2}_{n \ (K_1 K_2) \text{ blocks}} x$$

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Cor: Let $\mathcal{L}_{K,\neg}$ be the language generated by $\{K_1, K_2, \neg\}$. Then there are uncountably many levels of $\mathcal{L}_{K,\neg}$-information.

So what about conjunctions and disjunctions

Lemma

Let $\mathcal{L}_{K,\wedge}$ be the language generated by $\{K_1, \ldots, K_n, \wedge\}$, i.e. containing all formulas of the form

$$K_1(x \wedge K_2 K_3(x \wedge K_1 x))$$

Then there are only countably many levels of $\mathcal{L}_{K,\wedge}$-information.

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Then there are only countably many levels of $\mathcal{L}_{K,\wedge}$-information.

Let $D_J\varphi := \bigvee_{i \in J} K_i\varphi$, i.e. D is some sort of distributed knowledge.

Lemma

Let $\mathcal{L}_D$ be the reasoning language defined by $\{D_J \mid J \subseteq I\}$. Then $\mathcal{L}_D$ has only countably many levels of information.

More disjunctions

Lemma

Let $\mathcal{L}_{\vee 2}$ be the language generated by $\{K_1, K_2, \vee\}$, i.e. containing all formulas of the form

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Then $\mathcal{L}_{\vee 2}$ has only countably many levels of information.

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Then $\mathcal{L}_{\vee 2}$ has only countably many levels of information.

Lemma

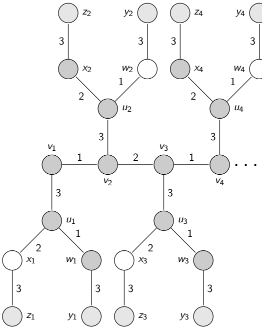
Let $\mathcal{L}_{K,\vee}$ be the language generated by $\{K_1, \ldots, K_n, \vee\}$ for $n \geq 3$. Then $\mathcal{L}_{K,\vee}$ has uncountably many levels of information.

The Counter Model

Define operators $B_1\varphi$ and $B_2\varphi$ as

$$B_1\varphi := K_1\left(K_3K_1x \vee \varphi\right) \qquad B_2\varphi := K_2\left(K_3K_2x \vee \varphi\right)$$

Then all formulas of the form $B_1B_2B_1\ldots\chi$ are mutually independent, where $\chi = K_3(K_1K_3x \vee K_2K_3x)$



Collecting Insights

The following languages have countably many levels of information:

Reasoning language	generated by
$\mathcal{L}_K$	$\{K_1 \dots K_n\}$ (Parikh/Krasucki)
$\mathcal{L}_L$	$\{L_1 \dots L_n\}$
$\mathcal{L}_{K,\wedge}$	$\{K_1, \dots, K_n, \wedge\}$
$\mathcal{L}_D$	$\{D_J \mid J \subseteq I\}$ where $D_J\varphi := \bigvee_{i\in J} K_i\varphi$
$\mathcal{L}_{\vee 2}$	$\{K_1, K_2, \vee\}$

ii) The following languages have uncountably many levels of info.:

Reasoning language	generated by
$\mathcal{L}_B$	$\{B_1 \dots B_n\}$
$\mathcal{L}_{L,K}$	$\{K_1, \dots, K_n, L_1 \dots L_n\}$
$\mathcal{L}_{K,\neg}$	$\{K_1 \dots K_n, \neg\}$
$\mathcal{L}_J$	$\{J_1, \dots, J_n\}$ where $J_i\varphi = K_i\varphi \vee K_i\neg\varphi$ (knowing whether, Hart *et al.*)
$\mathcal{L}_{K,\vee}$	$\{K_1, \dots, K_n, \vee\}$ for $n \geq 3$

The Question of Realizability

- Level of information as Goal State
- Is it realizable in a finite model?
- How to bring it about?

The Second Question: Realizing levels of Information

Definition

Let $\mathcal{L}_{res}$ be a reasoning language and $T \subseteq \mathcal{L}_{res}$ a level of information. We say that a Kripke model M, w realizes T iff for $\varphi \in \mathcal{L}_{res}$:

$$M, w \models \varphi \text{ iff } \varphi \in T.$$

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The big Question:

When is a level of information realizable in a finite model

Theorem

Let $\mathcal{L}_c$ be any of the reasoning languages we identified as having countably many levels and let T be a level of $\mathcal{L}_c$ information. Then T is realizable in a finite model.

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- For cardinality reasons, the result can't hold for reasoning languages allowing for uncountably many levels of information
- "Classic tradeoff between expressive power and realizability"

Proof Sketch

- Let T be a level for one of these reasoning languages
- Have seen: Level is characterized by finitely many minimal elements of the complement
- Take any (locally finite) model M, w realizing T
- Show: Can cut all parts far away from M while leaving informatioal level untouched

The Third Question: Learning new Things

Information changes

- Reasoning
- Private Communication
- Public announcements
- ...

But what does this entail about levels of information?

The Change of Information

- Only interested in information (no factual changes in the world)
- For now: Only interested in knowledge
- Two questions:
  - Potential developments of given level of information
  - Given a situation and a goal level of information: When and how can it be reached?

Representing Information Change

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$$\mathcal{M} \oplus E$$

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Initial epistemic model.

Abstract description of an epistemic event.

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- Public Announcements
- Private Communication
- Communication with (un)certain Success

Product Update Details

Let $\mathbb{M} = \langle W, (R_i), V \rangle$ be a Kripke model.

An event model is a tuple $\mathbb{A} = \langle A, (S_i), Pre \rangle$, where $S \subseteq A \times A$ is an equivalence relation and $Pre : A \to \mathcal{L}$.

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The update model $\mathbb{M} \oplus \mathbb{A} = \langle W', (R_i'), V' \rangle$ where

- $W' = \{(w, a) \mid w \models Pre(a)\}$
- $(w, a) R_i' (w', a')$ iff $w R_i w'$ and $a S_i a'$
- $(w, a) \in V(p)$ iff $w \in V(p)$

The Dynamics of Information

Theorem

Let T_1 and T_2 be levels of $\mathcal{L}_K$ information, Let $M(T_1)$ and $M(T_2)$ denote the minimal elements of the complement of T_1 and T_2.

i) There is a model M, w realizing T_1 and product model $\mathcal{E}, e$ such that $M, w \oplus \mathcal{E}, e$ realizes T_2 iff $T_1 \subseteq T_2$

The Dynamics of Information

Theorem

Let T_1 and T_2 be levels of $\mathcal{L}_K$ information, Let $M(T_1)$ and $M(T_2)$ denote the minimal elements of the complement of T_1 and T_2.

i) There is a model M, w realizing T_1 and product model $\mathcal{E}, e$ such that $M, w \oplus \mathcal{E}, e$ realizes T_2 iff $T_1 \subseteq T_2$

ii) There is a model M, w realizing T_1 be given. Then there is a product model $\mathcal{E}, e$ such that $M, w \oplus \mathcal{E}, e$ realizes T_2 if for all $\varphi \in M(T_2)$ there is $\psi \in M(T_1)$:

- *i)* $\psi \preceq \varphi$
- *ii)* *Let $\varphi = K_{i_1} \dots K_{i_r} x$ and $\psi = K_{j_1} \dots K_{j_s} x$. Then $K_{i_r} = K_{j_s}$.*

The Main Lessons

- Subtle changes can impact expressive power drastically
- Classic tradeoff between expressive power and realizability
- Realizing through public announcements or private communication

Some Potential Applications

- Information Dynamics on Social Networks
- The Emergence of Social Norms
- Cryptography Protocols

Also in the Thesis

- Logic and Reasoning in Games
- Logic and the Decision to Vote
- Non-logical models (of Expert Judgment and the Emergence of Trust)

Available at http://tinyurl.com/PhDSocialInteraction