Computing Siegel Modular Forms: Everything but Algebra

Abstract:
Siegel modular forms are highly symmetric functions, which, for example, are connected by the Langlands conjecture to Galois representations. Their Fourier expansions are important in various areas, including combinatorics and enumerative geometry. It had been a natural question from early on to compute these Fourier coefficients.

It is well-known how to consider Siegel modular forms as sections of certain vector bundles over "Siegel modular varieties". This interpretation has governed the computational theory until recently. It is based on Manin's initial work on modular symbols, which has stimulated tremendous progress. As a result, by today we have, for example, Cremona's database of elliptic curves, containing thousands of them.

In the last year, two novel approaches made appearance: One based on combinatorial properties of Fourier expansions by myself, another one based on PDEs by Monien. We will survey both, and discuss perspectives and current progress on implementations.