Quantum cohomology of flag manifolds and the Toda lattice

Abstract:
A finite number of particles move on a straight line such that each particle is connected with the next one by an ‘exponential' spring: in the 1960s physicists, most notably M. Toda, observed that the differential equation system that describes the motion of this lattice has certain simple features. The system received the name of Toda lattice. Mathematicians became interested in this topic after H. Flaschka realized in the mid 1970s that the Toda lattice is one of the few Hamiltonian mechanical systems that admit `sufficiently many' conserved quantities. Twenty years after that, the rigorous foundations of another theory inspired by physics were about to be established: the quantum cohomology. A challenging problem was to calculate this new geometric invariant for basic spaces, such as the complex projective line and its natural generalization, the full complex flag manifold. The description obtained by A. Givental and B. Kim in 1995 surprised everyone at that time and a veil of mystery persists around it even today: it says that the quantum cohomology ring of the flag manifold is determined by the conserved quantities of the Toda lattice. In the talk I will give more details concerning this connection. Two kinds of generalizations will also be discussed: Lie theoretical and infinite dimensional.

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